OPERATIONS RESEARCH

SUBJECT CODE: 18UAD9

Objectives

To helps students understand scientific method used in operation research

Unit – I

Introduction to OR : Scientific method – O.R.Models and Model Building – Resources Allocation – Linear Programming – Graphical Method – Simplex Method – M-Technique (Duality in Linear Programming Problem Excluded)

Unit –II

Transportation , Assignment & Sequencing : Transportation & Assignment Models – Sequential Decision Making – Sequencing Problems.

Unit – III

Inventory Management : Inventory Management – Deterministic and elementary stochastic Models.

Unit – IV

Simulation & Queuing : Simulation, Queuing Models (M/M/I)

Unit – V

Network Analysis : PERT & CPM – Replacement Decisions.

Outcome : To facilitate quantitative solution in business risk & uncertainity

REFERENCES

Operation research	-	Hamdy A.Taha
Operation Research Problem and Solutions	-	V.K.Kapoor
Operation Research	-	Gupta, Ganti Swroop & Mon Mohan.
Operations Research	-	Dr.P.R .Vittal & V. Malini

(Unit –I, Chapter-1,2,3,4,5; Unit –II, Chapter-,10,11,12; Unit-III, Chapter-17; Unit –IV,Chapter-13,18; Unit -V ,Chapter-14,16)

OPERATIONS RESEARCH

UNIT I

INTRODUCTION

Operations research is a new branch of mathematics dealing in the optimization problems in real-time situations. It is also a quantitative technique to deal many management problems. In this discipline we study cost minimization of various inventory problems, the minimization of transportation cost of sending goods from various warehouses to different centre's, the profit maximization or cost reduction in linear programming models, the assignment of different persons to different jobs so that total time taken to perform the jobs is minimized, the congestion problem in traffic places, supermarkets, airline counters, to find out the waiting tome of customers in the queue, the project completion time with limited resources and many other similar problems.

HISTORY OF OPERATIONS RESEARCH

The origin of the concept operations research occurred during the world war I. in England in the year 1915,F.W.Lanchester attempted to treat military operations quantitatively. He derived equations relating to the outcome o\f a battle to both the relative numerical strength of the combatants and their relative man power. He modeled a situation involving strategic choices and ten tested that model against a known situation.

In 1917, A.K. Erlang, a Danish mathematician has developed solutions for some waiting line problems. In 1930 W.Leontieff developed a linear programming model representing the entire United States economy. Active research works were done during World war II in great Britain and United States of America.

Two important developments in OR after world war II are worth noting. In 1947, George B.Dantizing developed simplex solution to linear programming problem. In 1958, network models were developed by U.S. special project office.

DEFINITIONS OF OPERATIONS RESEARCH

OR can be defined as:

- Operations Research is a scientific method of providing executive departments with a quantitative basis for decision regarding the operations under their control. -- P M Morse & GE & Kimball.

- Operations Research is a scientific approach to problem solving for executive Management. -- H.M Wagner
- OR is the art of giving bad answers to problems where otherwise worse answers were given." ---T.L.Sastry
- Operations research is defined as the art of winning wars without actually fighting."--- D'Clarke
- OR is the application of the methods of science to complex problem arising in the direction and management of large system of men, machines, materials and money in industry, business and government of defense. The distinctive approach is to develop a scientific model of the system, incorporating measurements of factors such as chance & risk, with which to predict and compare the outcome of alternative decisions strategies and controls. The purpose is to help management determine its policies and actions scientifically. — Journal of OR Society of UK
- OR is an experimental and applied science devoted to observing, understanding and predicting the behaviour of purposeful man-machine systems; and OR workers are actively engaged in applying this knowledge to practical problems in business, government and society" — OR Society of America

CHARACTERISTICS OF OR

1. Interdisciplinary team approach – The problems an OR of analyst faces are heterogeneous in nature, involving the number Variables and constraints which are beyond the analytical ability of a person. So a number of people from various disciplines are required to understand the problem. They apply their specialized knowledge and experience to get a better understanding and solution to the problem on hand.

   System approach – Any organization is it a business or government of a defence organization can be considered as a system having various sub systems. The decision made by any sub systems made by will have its effect on other sub systems. Like decision taken by finance department will have its effect on marketing department. When dealing with OR problem the system should be treated as a whole so that the interrelation between sub systems and the problem on the entire system are kept in mind. Hence OR is a system approach.

2. Scientific method – OR uses scientific methods for the following steps:

    - The problem is defined and analyzed
    - Observations are made under different conditions.
    - On the basis of observations, a hypothesis is formulated how the various factors interact for the best solution to the problem.
    - An experiment is designed and executed to test the hypothesis.
    - Finally the results of the experiments are analyzed and the hypothesis is either accepted or rejected.

3. It helps increasing the creative ability of the decision market – OR provides the managers mathematical took, techniques and models to analyze the problem on hand and to evaluate the result of all alternatives and make lair Optimal choice, thereby helping him in faster and better decisions. Hence a manager who uses OR techniques will have a better creativity ability than a manager who does not use these techniques.

4. Helpful in finding: optimum decisions – OR techniques always try to provide the best or optimum decisions regarding to the organization. It provides the solution by considering all the constraints.

5. Quantitative solutions – OR techniques provide quantitative basis for decision making to the management. Different problems related to business and management like Assignment problem, Transportation problem, Game Theory, Simulation, and Markov Chain etc. are solved in quantitative form.

6. Use of computer – Since OR techniques are mathematical in nature therefore it requires a computers to solve the complex mathematical models. A large amount of calculations are required so use of digital computer has become an integral part of the Operations research approach to decision making.

SIGNIFICANCE OF OPERATIONS RESEARCH

- The problem may be complex for which normal guess does not work. OR techniques help in arriving at a feasible solution to the problem that too with full analysis and research.
- The problem is very important and no lethargy on part of the decision-maker can be entertained. In such a situation also, OR comes to rescue. OR methodologies are so detailed and meticulous that even a little scope for leaving a point unattended is unaffordable.
- The problem is new and no precedence exists to facilitate logical and intelligent decision making of the problem. In this situation, OR comes to the rescue of analysts and the reason for this is that OR has a large pool of techniques that aid to solve even new and unthought-of problems.
- The problem is repetitive and a model may be developed to enhance faster and better decisions. Again OR helps like nothing else to aid the analysts in such a situation by offering either a readymade model, or a readymade process to develop such a model and pacify the decision making process.

SCOPE OF OPERATIONS RESEARCH

Operations Research addresses a wide variety of issues in transportation, inventory planning, production planning, communication operations, computer operations, financial assets, risk management, revenue management, and many other fields where improving business productivity is paramount. OR supports the key decision making process, allows to solve urgent problems, can be utilized to design improved multi-step operations (processes), setup policies, supports the planning and forecasting steps, and measures actual results.

- Manufacturing: OR's success in contemporary business pervades manufacturing and service operations, logistics, distribution, transportation, and telecommunication.

- Revenue Management: The application of OR in revenue management entails first to accurately forecasting the demand, and secondly to adjust the price structure over time to more profitably allocate fixedϖ capacity.
- Supply Chain Management: In the area of Supply Chain Management, OR helps in taking decisions that include the who, what, when, and where abstractions from purchasing and transporting raw materials and parts, through manufacturing actual products and goods, and finally distributing and delivering the items to the customers.
- Retailing: In supermarkets, data from store loyalty card schemes is analyzed by OR groups to advice on merchandising policies and profitability improvement.
- Financial Services: In financial markets, OR practitioners address issues such as portfolio and risk management and planning and analysis of customer service. They are widely employed in Credit Risk Management—a vital area for lenders needing to ensure that they find the optimum balance of risk and revenue.
- Marketing Management: OR helps marketing manager in making the apt selection of product mix. It helps them in making optimum sales resource allocation and assignments.
- Human Resource Management: OR techniques are being applied widely in the functional area of Human Resource Management by helping the human resource managers in activities like manpower planning, resource allocation, staffing and scheduling of training programs.
- General Management: OR helps in designing Decision Support System and management of information systems, organizational design and control, software process management and Knowledge Management.
- Production systems: The area of operations research that concentrates on real-world operational problems is known as production systems. Production systems problems may arise in settings that include, but are not limited to, manufacturing, telecommunications, health-care delivery, facility location and layout, and staffing.

MODELS OF OPERATIONS RESEARCH

When a problem or process under investigation is simplified and represented with its typical features or characteristics, it is called as a model. The word 'model' has several meanings. For example, a model can act as a substitute for representing reality, such as small scale model locomotive or may act as some sort of idealization, like a model plan for recruitment procedure, etc. Constructing a model helps in bringing the complexities and possible uncertainties into a logical framework required for comprehensive analysis. In fact the model acts as a vehicle in arriving at a well-structured view of reality. An array of models can be seen in various business areas or industrial activities. The most relevant for our study purpose are portrayed below:

1. Physical Models:
   - (a) Iconic Models
   - (b) Analogue Models
2. Symbolic Models:
   - (a) Verbal Models
   - (b) Mathematical Models:
     - (i) Deterministic Models
     - (ii) Probabilistic Model

Physical Models: To deal with specific types of problems, models like diagrams, charts, graphs and drawings are used, which are known as "Physical Models'. The schematic way of representation of significant factors and interrelationship may be in a pictorial form help in useful analysis. Moreover, they help in easy observation, description and prediction.

There are two types of Physical Models:

(1) Iconic Models: An image or likeness of an object or process is Icon. These models represent the system as it is by scaling it up or down. Even though use of these models in the area of management appears to be narrow, their usefulness is seen in the field of engineering and science. For example,

(a) In the field of R&D, prototype of the product is developed and tested to know the Work ability of the new product development.

(b) Photographs, portraits, drawings are the good example of iconic types. These models help in testifying the samples thus avoiding full scale designing and probable loss.

Analogue Models: These models are closely associated with iconic models. However, they are not the replicas of system or process. The analogue, in constructing these models, help in analyzing the issues and forces which are in the system or process. Because, these models use 'one set of properties' which is 'analogous to another set of properties.' For example, kids, toys, rail-road models, etc.

(2) Symbolic Models: Symbolic Models use letters, numbers, figures to represent decision variables of the system. There are two types of Symbolic Models—Verbal Models and Mathematical Models.

(a) Verbal Models: These models describe a situation in written or spoken language. Written sentences, books, etc., are examples of a verbal model.

(b) Mathematical Models: Mathematical symbols are used to represent a problem or a system under these types of models. Rules of mathematics enable the builder to make the models more abstract and precise. There are two types of Mathematical Models—Deterministic Models and Probabilistic Models.

(i) Deterministic Models: The exact statement of variables and their relationships are made under these models. The coefficients used for the mathematical formulation are known and are constant with certainty. So, to say, with a given set of data the answer will always be the same. For instance, determination of the break-even sales volume (BEP), the volume where the total cost equals the total sales revenue earned pertaining to a product.

(ii) Probabilistic Models: The risk involved and the state of uncertainty are covered by these models. The decision variables take the form of a probability distribution and can assume more than single values. In the presence of risk and uncertainty, these models tend to yield different answers every time when attempted to.

LINEAR PROGRAMMING

Linear Programming Problem (LPP) is a mathematical technique which is used to optimize (maximize or minimize) the objective function with the limited resources.

Mathematically, the general linear programming problem (LPP) may be stated as follows.

Maximize or Minimize $Z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$

Subject to the conditions (constraints)

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq (or = or \geq) b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq (or = or \geq) b_2$$

$$\ldots \quad \ldots. \quad \ldots. \quad \ldots. \quad \ldots$$

$$\ldots \quad \ldots. \quad \ldots. \quad \ldots. \quad \ldots$$

$$a_{m1}x1 + a_{m2}x_2 + \ldots + a_{mn}x_n (\leq or = or \geq) b_m$$

$$x_1, x_2 \ldots\ldots. x_n \geq 0$$

Short form of LPP

$$\text{Maximize or Minimize} \qquad Z = \sum_{j=1}^{n} c_j x_j$$

$$\text{Subject to} \sum_{j=1}^{n} a_{ij} x_j \leq (or = or \geq) b_i \text{, i} = 1,2,3,\ldots,m \qquad \ldots (1)$$

$$\text{and} \quad x_j \geq 0 \qquad \ldots (2)$$

BASIC TERMINOLOGIES

The word 'linear' is used to describe the relationship among two or more variables which are directly or precisely proportional. Programming' means the decisions which are taken systematically by adopting alternative courses of action.

1. Decision Variables and their Relationships: The decision variable refers to any candidate (person, service, projects, jobs, tasks) competing with other decision variables for limited resources. These variables are usually interrelated in terms of utilization of resources and need simultaneous solutions, i.e., the relationship among these variables should be linear.
2. Objective Function: The Linear Programming Problem must have a well defined objective function to optimize the results. For instance, minimization of cost or maximization of profits. It should be expressed as linear function of decision variables (Z = X1 + X2 , where Z represents the objective, i.e., minimization/maximization, X1 and X2 are the decision variables directly affecting the Z value).
3. Constraints: There would be limitations on resources which are to be allocated among various competing activities. These must be capable of being expressed as linear equalities or inequalities in terms of decision variables.

4. Alternative Courses of Action: There must be presence of alternative solutions for the purpose of choosing the best or optimum one

5. Non-Negativity Restrictions: All variables must assume non-negative values. If any of the variable is unrestricted in sign, a tool can be employed which will enforce the negativity without changing the original information of a problem.

6. Linearity and Divisibility: All relationships (objective function and constraints) must exhibit linearity i.e., relationship among decision variables must be directly proportional. It is assumed that decision variables are continuous, i.e., fractional values of variables must be permissible in obtaining the optimum solution.

7. Feasible solution: A set of values of the decision variables that satisfies all the constraints of the problem and non-negativity restrictions is called a feasible solution of the problem.

8. Optimal solution: Any feasible solution which maximizes or minimizes the objective function is called an optimal solution.

9. Feasible region: The common region determined by all the constraints including non-negative Constraints $x_j \geq 0$ of a linear programming problem is called the feasible region (or solution region) for the problem.

10. Deterministic: In Linear Programming it is assumed that all model coefficients are completely known. For example: profit per unit.

ADVANTAGES AND LIMITATIONS OF OR

ADVANTAGES	LIMITATIONS
It helps in proper and optimum utilization of the scarce resources	The treatment of variables having nonlinear relationships is the greatest limitation of this LP
It helps in improving the quality of the decisions.	It can come out with non-integer solutions too, which would be many times meaningless.

With the use of this technique, the decision-maker becomes more objective and less subjective	It rules out effect of time and uncertainty conditions.
It even helps in considering other constraints operating outside the problem.	Generally, the objective set will be single and on the contrary, in the real life, there might be several objectives
Many a times it hints the manager about the hurdles faced during the production activities.	Large-scale problems tend to be unaccomodative to solve under LP

Formulation of LP Models

Steps for Formulating LPP

1. Identify the nature of the problem (maximization/minimization problem).
2. Identify the number of variables to establish the objective function.
3. Formulate the constraints.
4. Develop non-negativity constraints.

Q.No:1

A company is producing three products P_1, P_2 and P_3, with profit contribution of Rs.20, Rs.25 and Rs.15 per unit respectively. The resource requirements per unit of each of the products and total availability are given below.

Product	P_1	P_2	P_3	Total availability
Man hours/unit	6	3	12	200
Machine hours/unit	2	5	4	350
Material/unit	1kg	2kg	1kg	100kg

Formulate the above as a linear programming model.

Solution:

i) Variables: Let x_1 , x_2 and x_3 be the number of units of products P_1, P_2 and P_3 to be produced.

(ii) Objective function: Profit on x_1 units of the product $P_1 = 20\ x_1$

Profit on x_2 units of the product $P_2 = 25\ x_2$

Profit on x_3 units of the product $P_3 = 15\,x_3$

Total profit = $20\,x_1 + 25\,x_2 + 15\,x_3$

Since the total profit is to be maximized, we have to maximize $Z = 20\,x_1 + 25\,x_2 + 15\,x_3$

Constraints:

$6x_1 + 3\,x_2 + 12\,x_3 \leq 200$

$2\,x_1 + 5\,x_2 + 4\,x_3 \leq 350$

$x_1 + 2\,x_2 + x_3 \leq 100$

Non-negative restrictions: Since the number of units of the products A,B and C cannot be negative, we have $x_1, x_2, x_3 \geq 0$

Thus, we have the following linear programming model.

Maximize $Z = 20\,x_1 + 25\,x_2 + 15\,x_3$

Subject to

$6\,x_1 + 3\,x_2 + 12\,x_3 \leq 200$

$2x_1 + 5x_2 + 4\,x_3 \leq 350$

$x_1 + 2\,x_2 + x_3 \leq 100$

$x_1, x_2, x_3 \geq 0$

Q.No:2

A dietician wishes to mix two types of food F_1 and F_2 in such a way that the vitamin contents of the mixture contains atleast 6units of vitamin A and 9 units of vitamin B. Food F_1 costs Rs.50 per kg and F_2 costs Rs 70 per kg. Food F_1 contains 4 units per kg of vitamin A and 6 units per kg of vitamin B while food F_2 contains 5 units per kg of vitamin A and 3 units per kg of vitamin B. Formulate the above problem as a linear programming problem to minimize the cost of mixture.

Solution:

(i) Variables: Let the mixture contains x_1 kg of food F_1 and x_2 kg of food F_2

(ii) Objective function:

Cost of x_1 kg of food $F_1 = 50\,x_1$

Cost of x_2 kg of food $F_2 = 70x_2$

The cost is to be minimized

Therefore minimize $Z = 50x_1 + 70x_2$

Resources	Food (in kg)		Requirement
	F_1 (x_1)	F_2 (x_2)	
Vitamin A (units/kg)	4	5	6
Vitamin B (units/kg)	6	3	9
Cost (Rs/kg)	50	70	

(iii) Constraints:

$4x_1+5x_2 \geq 6$ (since the mixture contains 'atleast 6' units of vitamin A , we have the inequality of the type $\geq$)

$6x_1+3x_2 \geq 9$(since the mixture contains 'atleast 9' units of vitamin B ,we have the inequality of the type $\geq$)

(iv) Non-negative restrictions:

Since the number of kgs of vitamin A and vitamin B are non-negative, we have $x_1, x_2 \geq 0$

Thus, we have the following linear programming model

Minimize $Z = 50x_1 + 70x_2$

subject to $4x_1+5x_2 \geq 6$

$6x_1+3x_2 \geq 9$

and $x_1, x_2 \geq 0$

LINEAR PROGRAMMING- GRAPHICAL METHOD

After formulating the linear programming problem, our aim is to determine the values of decision variables to find the optimum (maximum or minimum) value of the objective function. Linear programming problems which involve only two variables can be solved by graphical method. If the problem has three or more variables, the graphical method is impractical.

The major steps involved in this method are as follows

(i) State the problem mathematically

(ii) Write all the constraints in the form of equations and draw the graph

(iii) Find the feasible region

(iv) Find the coordinates of each vertex (corner points) of the feasible region. The coordinates of the vertex can be obtained either by inspection or by solving the two equations of the lines intersecting at the point

(v) By substituting these corner points in the objective function we can get the values of the objective function

(vi) If the problem is maximization then the maximum of the above values is the optimum value. If the problem is minimization then the minimum of the above values is the optimum value

Q.No:3

Solve the following LPP

Maximize $Z = 2x_1 + 5x_2$

subject to the conditions $x_1 + 4x_2 \leq 24$

$3x_1 + x_2 \leq 21$

$x_1 + x_2 \leq 9$ and $x_1, x_2 \geq 0$

Solution:

First we have to find the feasible region using the given conditions.

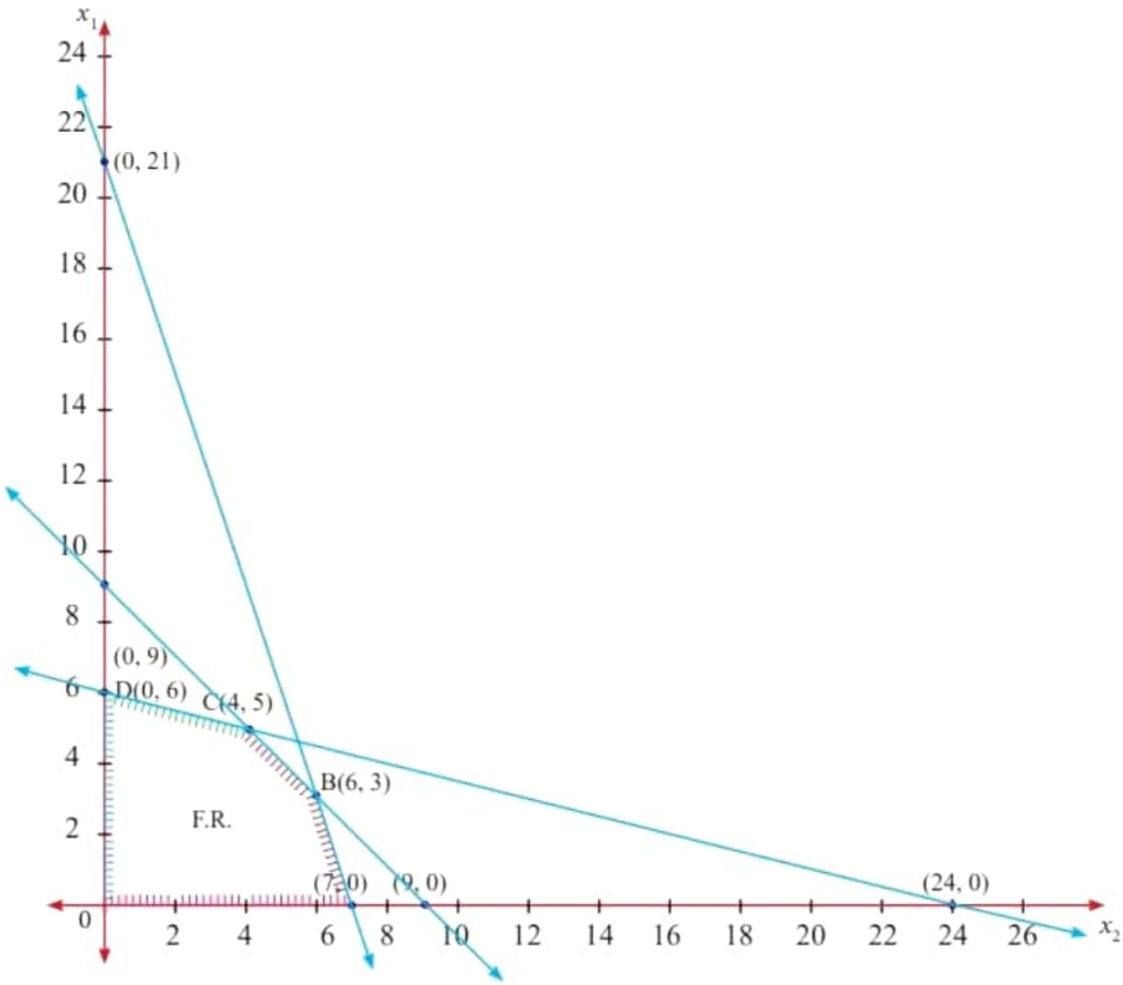
Since both the decision variables x_1 and x_2 are non-negative, the solution lies in the first quadrant. Write all the inequalities of the constraints in the form of equations.

Therefore we have the lines $x_1 + 4x_2 = 24$; $3x_1 + x_2 = 21$; $x_1 + x_2 = 9$ $x_1 + 4x_2 = 24$ is a line passing through the points (0, 6) and (24, 0). [(0, 6) is obtained by taking $x_1 = 0$ in $x_1 + 4x_2 = 24$, (24, 0) is obtained by taking $x_2 = 0$ in $x_1 + 4x_2 = 24$].

Any point lying on or below the line $x_1 + 4x_2 = 24$ satisfies the constraint $x_1 + 4x_2 \leq 24$.

$3x_1 + x_2 = 21$ is a line passing through the points (0, 21) and (7, 0). Any point lying on or below the line $3x_1 + x_2 = 21$ satisfies the constraint $3x_1 + x_2 \leq 21$.

$x_1 + x_2 = 9$ is a line passing through the points (0 , 9) and (9 , 0) .Any point lying on or below the line $x_1 + x_2 = 9$ satisfies the constraint $x_1 + x_2 \leq 9$.



The feasible region satisfying all the conditions is OABCD. The co-ordinates of the points are O(0,0) A(7,0);B(6,3) [the point B is the intersection of two lines $x_1+ x_2= 9$ and $3\,x_1+ x_2= 21$];C(4,5) [the point C is the intersection of two lines $x_1+ x_2 = 9$ and $x_1+ 4x_2 = 24$] and D(0,6).

Corner points	$Z = 2\,x_1 + 5x_2$
O(0,0)	0
A(7,0)	14
B(6,3	27
C(4,5)	33
D (0, 6).	30

Maximum value of Z occurs at C. Therefore the solution is $x_1 = 4$, $x_2 = 5$, $Z_{max} = 33$

Q.No:4

Solve the following LPP by graphical method Minimize $z = \mathbf{5}x_1+\mathbf{4}x_2$ Subject to constraints $4x_1+ x_2 \geq 40$; $2x_1+3x_2 \geq 90$ and $x_1, x_2 \geq 0$

Solution:

$4x_1+x_2 = 40$ is a line passing through the points (0, 40) and (10, 0).Any point lying on or above the line $4x_1+x_2= 40$ satisfies the constraint $4x_1+ x_2 \geq 40$.

$2x_1+3x_2 = 90$ is a line passing through the points (0, 30) and (45, 0). Any point lying on or above the line $2x_1+3x_2= 90$ satisfies the constraint $2x_1+3x_2 \geq 90$.

Draw the graph using the given constraints.

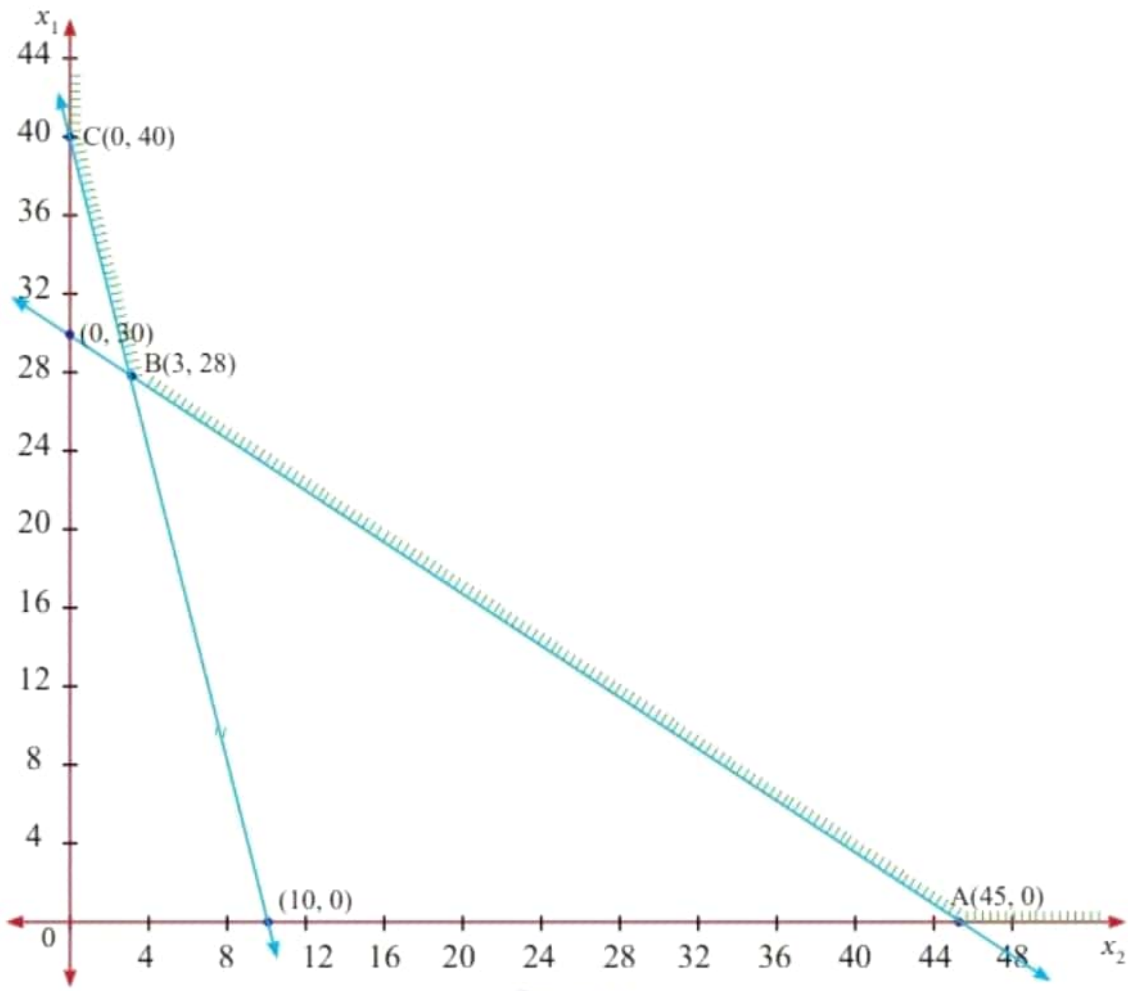


Fig 10.2

The feasible region is ABC (since the problem is of minimization type we are moving towards the origin.

Corner points	$z = 5x_1+4x_2$
A(45,0)	225
B(3,28)	127
C(0,40)	160

The minimum value of Z occurs at B (3, 28).

Hence the optimal solution is $x_1 = 3$, $x_2 = 28$ and $Z_{min}=127$

Q.No:5

Solve the following LPP.

Maximize $Z= 2x_1 +3x_2$

subject to constraints $x_1 + x_2 \leq 30$; $x_2 \leq 12$; $x_1 \leq 20$ and $x_1, x_2 \geq 0$

Solution:

We find the feasible region using the given conditions.

Since both the decision variables x_1 and x_2 are non-negative, the solution lies in the first quadrant of the plane.

Write all the inequalities of the constraints in the form of equations.

Therefore we have the lines

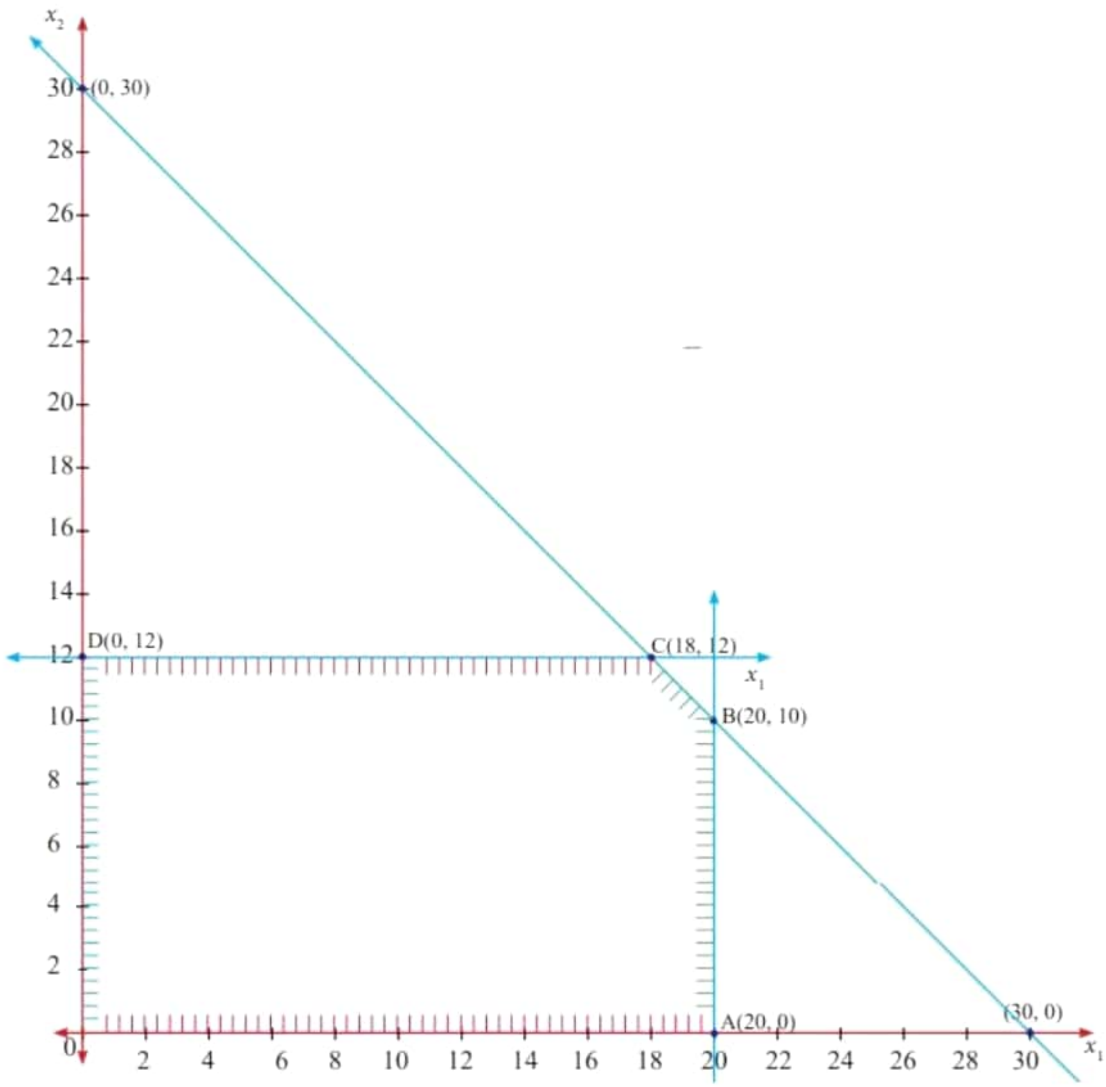
$x_1+x_2=30$; $x_2 =12$; $x_1= 20$

$x_1+x_2 =30$ is a line passing through the points (0,30) and (30,0)

$x_2 = 12$ is a line parallel to x_1–axis

$x_1 = 20$ is a line parallel to x_2–axis.

The feasible region satisfying all the conditions $x_1+ x_2\leq 30$; $x_2\leq 12$; $x_1\leq 20$ and $x_1, x_2 \geq 0$ is shown in the following graph.



The feasible region satisfying all the conditions is OABCD.
The co-ordinates of the points are O(0,0) ; A(20,0); B(20,10) ; C(18,12) and D(0,12).

Corner points	$Z = 2x_1 + 3x_2$
O(0,0)	0
A(20,0)	40
B(20,10)	70
C(18,12)	72
D(0,12)	36

Maximum value of Z occurs at C. Therefore the solution is $x_1 = 18$, $x_2 = 12$, $Z_{max} = 72$

Q.No:6

Maximize $Z = 3x_1 + 4x_2$

subject to $x_1 - x_2 \leq -1$; $-x_1+x_2 \leq 0$ and $x_1, x_2 \geq 0$

Solution:

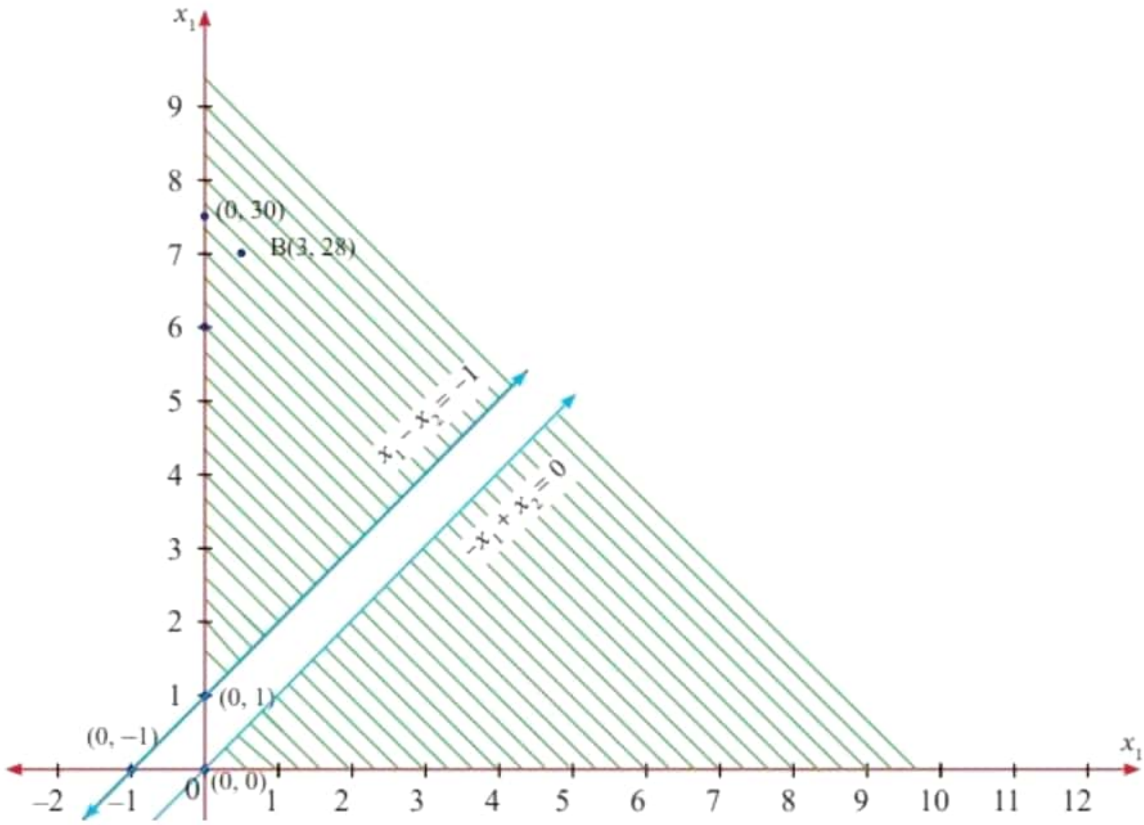
Since both the decision variables x_1, x_2 are non-negative ,the solution lies in the first quadrant of the plane.

Consider the equations $x_1 - x_2 = -1$ and $-x_1 + x_2 = 0$

$x_1 - x_2 = -1$ is a line passing through the points (0,1) and (−1,0)

$-x_1 + x_2 = 0$ is a line passing through the point (0,0)

Now we draw the graph satisfying the conditions $x_1 - x_2 \leq -1$; $-x_1+x_2 \leq 0$ and $x_1, x_2 \geq 0$



There is no common region (feasible region) satisfying all the given conditions.

Hence the given LPP has no solution.

LINEAR PROGRAMMING - SIMPLEX METHOD - MAXIMISATION

SIMPLEX METHOD

The simplex method is a technique of solving linear programming problems by obtaining a feasible solution and, by iterative procedure improving, this solution until the optimal solution is reached. The computational routine for the simplex method is based on matrix algebra and consists of obtaining an inverse matrix in order to solve a set of simultaneous linear equations.

Procedure

Step 1: Convert inequality constraints into equality constraints by introducing slack variables.

Step 2: Introduce zero co-efficient to the slack variables into the objective function.

Step 3: Prepare the simplex table. Take the initial solution as X1=0 and X2=0, so that Z=0. The simplex table consists of 3 components. The first component is the first 3 columns in the table-giving Current variable, contributions and quantities.

Step 4: The second component of the simplex table is called the body of the simplex table. Hence each row represents the coefficient of one constraint.

Step 5: At the top of the table write down Cj, the contributions of all the variables.

Step 6: Introduce 2 more rows; one for Zj and the other for Zj-Cj. Zj_s are obtained by multiplying each column elements with the corresponding contributions and adding.

Step 7: If Zj-Cj$\geq$0 the initial solution is the optimal solution. If this is not the case the least value in Zj-Cj decides the incoming variable to improve the solution.

Step 8: To decide the outgoing variable determines the ratio between the elements in the quantity column and the elements in the incoming variable column (3^{rd} component). The least positive ratio decides the outgoing variable. The element common to the incoming variable column and the outgoing variable row is called the pivotal element. We have to make this element unity in the nest simplex table.

Q.No:7

Solve the following problem using simplex method

Maximize $Z= 4x_1+7x_2$

Subject to the constraints

$4x_1+3x_2\leq12$

$3x_1+4x_2\leq12$

$x_1 , x_2 \geq0$

Solution

First convert the inequalities constraints into equality constraints

The LPP is maximize $Z=4x_1+7x_2+0\ S_1+0\ S_2$

Subject to $4x_1+3x_2+ S_1=12$

$3x_1+4x_2+ S_2=12$

$x_1 , x_2 , S_1, S_2\geq0$

Table 1:

		Cj	4	7	0	0	
Q	CV	C_j	X_1	X_2	S_1	S_2	Ratio
12	S_1	0	4	3	1	0	4
12	S_2	0	3	**4**	0	1	3 ←
		Zj	0	0	0	0	
		Zj -Cj	-4	-7 ▲	0	0	

Table 2:

		Cj	4	7	0	0	
Q	CV	C_j	X_1	X_2	S_1	S_2	Ratio
3	S_1	0	7/4	0	1	-3/4	R_1-$3R_2$
3	X_2	7	3/4	1	0	1/4	
		Zj	21/4	7	0	7/4	
		Zj -Cj	5/4	0	0	7/4	

Since all the Zj -Cj≥0, the optimal solution is reached. The optimal solution is X_1=0, X_2=3 and Max Z=21.

Q.No:8

Solve the following problem using simplex method

Maximize $Z= 21x_1+15x_2$

Subject to the constraints

$-X_1-2x_2 \geq -6$

$4x_1+3x_2 \leq 12$

$x_1 , x_2 \geq 0$

Solution

First convert the inequalities constraints into equality constraints

The LPP is maximize $Z=21x_1+15x_2+0\ S_1+0\ S_2$

Subject to $x_1+2x_2+ S_1=6$

$4x_1+3x_2+ S_2=12$

$x_1 , x_2 , S_1, S_2 \geq 0$

Table 1:

		Cj	21	15	0	0	
Q	CV	C_j	X_1	X_2	S_1	S_2	Ratio
6	S_1	0	1	2	1	0	6
12	S_2	0	**4**	3	0	1	3←
		Zj	0	0	0	0	
		Zj -Cj	-21 ↑	-15	0	0	

Table 2

		Cj	21	15	0	0	
Q	CV	C_j	X_1	X_2	S_1	S_2	Ratio
3	S_1	0	0	5/4	1	-1/4	R_1-R_2
3	X_2	21	1	3/4	0	1/4	
		Zj	21	63/4	0	21/4	
		Zj -Cj	0	3/4	0	21/4	

Since all the $Z_j -C_j \geq 0$, the optimal solution is reached. The optimal solution is $X_1=3$, $X_2=0$ and Max Z=63.

UNIT 2

TRANSPORTATION

This is a type of linear programming problem that may be solved using a simplified version of the simplex technique called transportation method. Because of its major application in solving problems involving several product sources and several destinations of products, this type of problem is frequently called the transportation problem. In a transportation problem, we have certain origins, which may represent factories where we produced items and supply a required quantity of the products to a certain number of destinations. This must be done in such a way as to maximize the profit or minimize the cost. Thus we have the places of production as origins and the places of supply as destinations. Sometimes the origins and destinations are also termed as sources and sinks.

Transportation model is used in the following:

- To decide the transportation of new materials from various centres to different manufacturing plants. In the case of multi-plant company this is highly useful.

- To decide the transportation of finished goods from different manufacturing plants to the different distribution centres. For a multi-plant-multi-market company this is useful.

MATHEMATICAL FORMULATION

Supposed a company has m warehouses and n retail outlets. A single product is to be shipped from the warehouses to the outlets. Each warehouse has a given level of supply, and each outlet has a given level of demand. We are also given the transportation cost between every pair of warehouse and outlet, and these costs are assumed to be linear. More explicitly, the assumptions are:

The total supply of the products from warehouse i = a i ,where i 1,2,3...m
The total Demand of the products at the outlet $j = j\ b$, where j 1,2,3,...n .
The cost of sending one unit of the product from warehouse i to outlet j is equal to
Cij , where i 1,2,3...m and j 1,2,3,...n . The total cost of a shipment is linear in size of shipment.

The Decision Variables

The variables in the Linear Programming (LP) model of the TP will hold the values for the number of units shipped from one source to a destination.

The decision variables are:

X_{ij} = the size of shipment from warehouse *i* to outlet *j* ,

Where *i* 1,2,3...*m* and *j* 1,2,3,...*n* .

This is a set of m.n variables.

The Objective Function

The objective function contains costs associated with each of the variables. It is a minimization problem.

Consider the shipment from warehouse *i* to outlet *j* . For any *i* and *j* , the transportation cost per unit *ij C* and the size of the shipment is *ij X* **.** Since we assume that the total cost function is linear, the total cost of this shipment is given by *c xij ij*

Summing over all *i* and j now yields the overall transportation cost for all warehouse-outlet combinations.

THE TRANSPORTATION TABLEAU

To Destination→ ↓From Source↓	D_1	D_2	...Dj...	D_n	Source Supply
S_1	c_{11} x_{11}	c_{12} x_{12}		c_{1n} x_{1n}	a_1
S_2	c_{21} x_{21}	c_{22} x_{22}		c_{2n} x_{2n}	a_2
... S_i ...			c_{ij} x_{ij}		... a_i ...
S_m	c_{m1} x_{m1}	c_{m2} x_{m2}		c_{mn} x_{mn}	a_m
Destination Requirements	b_1	b_2	... b_j ...	b_m	$\sum a_i$ $\sum b_j$

An optimal solution to a transportation problem is obtained in two stages.

Stage 1 is to obtain an initial solution.

Stage 2 is to obtain an optimal solution from the basic initial solution.

Initial feasible solution – Methods

1. North west corner rule.

2. Row minima method.

3. Column minima method.

4. Least cost entry method.

5. Vogel's approximation method.

North West Corner Rule:

Procedure:

Step-1: Select the upper left corner cell of the transportation matrix and allocate.

Step-2:a. Subtract this value from supply and demand of respective row and column.

b. If the supply is 0, then cross (strike) that row and move down to the next cell.

c. If the demand is 0, then cross (strike) that column and move right to the next cell.

d. If supply and demand both are 0, then cross (strike) both row & column and move diagonally to the next cell.

Step-3: Repeat this steps until all supply and demand values are 0

Find Solution using North-West Corner method

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	

Solution:

TOTAL number of supply constraints: 3

TOTAL number of demand constraints: 4

The rim values for S1=7 and D1=5 are compared.

The smaller of the two i.e. min(7,5) = 5 is assigned to S1 D1

This meets the complete demand of D1 and leaves 7 - 5 = 2 units with S1

Table 1:

	D_1	D_2	D_3	D_4	Supply
S_1	19**(5)**	30	50	10	2
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	0	8	7	14	

The rim values for S1=2 and D2=8 are compared.

The smaller of the two i.e. min(2,8) = 2 is assigned to S1 D2

This exhausts the capacity of S1 and leaves 8 - 2 = 6 units with D2

Table 2:

	D_1	D_2	D_3	D_4	Supply
S_1	19**(5)**	30**(2)**	50	10	0
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	0	6	7	14	

The rim values for S2=9 and D2=6 are compared.

The smaller of the two i.e. min(9,6) = 6 is assigned to S2 D2

This meets the complete demand of D2 and leaves 9 - 6 = 3 units with S2

Table3:

	D_1	D_2	D_3	D_4	Supply
S_1	19**(5)**	30**(2)**	50	10	0
S_2	70	30**(6)**	40	60	3
S_3	40	8	70	20	18
Demand	0	0	7	14	

The rim values for S2=3 and D3=7 are compared.

The smaller of the two i.e. min(3,7) = 3 is assigned to S2 D3

This exhausts the capacity of S2 and leaves 7 - 3 = 4 units with D3

Table 4:

	D_1	D_2	D_3	D_4	Supply
S_1	19**(5)**	30**(2)**	50	10	0
S_2	70	30**(6)**	40**(3)**	60	0
S_3	40	8	70	20	18
Demand	0	0	4	14	

The rim values for S3=18 and D3=4 are compared.

The smaller of the two i.e. min(18,4) = 4 is assigned to S3 D3

This meets the complete demand of D3 and leaves 18 - 4 = 14 units with S3

Table5 :

	D_1	D_2	D_3	D_4	Supply
S_1	19**(5)**	30**(2)**	50	10	0
S_2	70	30**(6)**	40**(3)**	60	0
S_3	40	8	70**(4)**	20	14
Demand	0	0	0	14	

The rim values for S3=14 and D4=14 are compared.

The smaller of the two i.e. min(14,14) = 14 is assigned to S3 D4

Table 6:

	D_1	D_2	D_3	D_4	Supply
S_1	19**(5)**	30**(2)**	50	10	0
S_2	70	30**(6)**	40**(3)**	60	0
S_3	40	8	70**(4)**	20**(14)**	0
Demand	0	0	0	0	

Initial feasible solution is

	D_1	D_2	D_3	D_4	Supply
S_1	19 **(5)**	30 **(2)**	50	10	7
S_2	70	30 **(6)**	40 **(3)**	60	9

S_3	40	8	70 **(4)**	20 **(14)**	18
Demand	5	8	7	14	

The minimum total transportation cost =19×5+30×2+30×6+40×3+70×4+20×14=1015

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6
∴ This solution is non-degenerate

ROW MINIMA METHOD

Procedure

Step-1: In this method, we allocate as much as possible in the lowest cost cell of the first row, i.e. allocate min(si,dj).

Step-2: a. Subtract this min value from supply si and demand dj.

b. If the supply si is 0, then cross (strike) that row and If the demand dj is 0 then cross (strike) that column.

c. If min unit cost cell is not unique, then select the cell where maximum allocation can be possible

Step-3: Repeact this process for all uncrossed (unstriked) rows and columns until all supply and demand values are 0.

Find Solution using Row minima method

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	

Solution:
TOTAL number of supply constraints : 3
TOTAL number of demand constraints : 4
Problem Table is

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9

S_3	40	8	70	20	18
Demand	5	8	7	14	

In 1st row, The smallest transportation cost is 10 in cell S1D4.

The allocation to this cell is min(7,14) = 7.

This exhausts the capacity of S1 and leaves 14 - 7 = 7 units with D4
Table 1:

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10**(7)**	0
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	7	

In 2nd row, The smallest transportation cost is 30 in cell S2D2.

The allocation to this cell is min(9,8) = 8.

This satisfies the entire demand of D2 and leaves 9 - 8 = 1 units with S2
Table 2:

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10**(7)**	0
S_2	70	30**(8)**	40	60	1
S_3	40	8	70	20	18
Demand	5	0	7	7	

In 2nd row, The smallest transportation cost is 40 in cell S2D3.

The allocation to this cell is min(1,7) = 1.

This exhausts the capacity of S2 and leaves 7 - 1 = 6 units with D3

Table 3:

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10(7)	0
S_2	70	30(8)	40(1)	60	0
S_3	40	8	70	20	18
Demand	5	0	6	7	

In 3rd row, The smallest transportation cost is 20 in cell S3D4.

The allocation to this cell is min(18,7) = 7.

This satisfies the entire demand of D4 and leaves 18 - 7 = 11 units with S3

Table 4:

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10(7)	0
S_2	70	30(8)	40(1)	60	0
S_3	40	8	70	20(7)	11
Demand	5	0	6	0	

In 3rd row, The smallest transportation cost is 40 in cell S3D1.

The allocation to this cell is min(11,5) = 5.

This satisfies the entire demand of D1 and leaves 11 - 5 = 6 units with S3

Table 5:

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10(7)	0
S_2	70	30(8)	40(1)	60	0
S_3	40(5)	8	70	20(7)	6
Demand	0	0	6	0	

In 3rd row, The smallest transportation cost is 70 in cell S3D3.

The allocation to this cell is min(6,6) = 6.

Table 6:

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10(7)	0
S_2	70	30(8)	40(1)	60	0
S_3	40(5)	8	70(6)	20(7)	0
Demand	0	0	0	0	

Initial feasible solution is

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10 (7)	7
S_2	70	30 (8)	40 (1)	60	9
S_3	40 (5)	8	70 (6)	20 (7)	18
Demand	5	8	7	14	

The minimum total transportation cost $=10\times7+30\times8+40\times1+40\times5+70\times6+20\times7=1110$

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6

∴ This solution is non-degenerate

COLUMN MINIMA METHOD

Procedure

Step-1: In this method, we allocate as much as possible in the lowest cost cell of the first Column, i.e. allocate min(si,dj).

Step-2: a. Subtract this min value from supply si and demand dj.

b. If the supply si is 0, then cross (strike) that row and If the demand dj is 0 then cross (strike) that column.

c. If min unit cost cell is not unique, then select the cell where maximum allocation can be possible

Step-3: Repeact this process for all uncrossed (unstriked) rows and columns until all supply and demand values are 0.

Find Solution using Column minima method

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9

S3	40	8	70	20	18
Demand	5	8	7	14	

Solution:
TOTAL number of supply constraints : 3
TOTAL number of demand constraints : 4

Problem Table is

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	

In 1st column, The smallest transportation cost is 19 in cell S1D1

The allocation to this cell is min(7,5) = 5.

This satisfies the entire demand of D1 and leaves 7 - 5 = 2 units with S1
Table 1:

	D_1	D_2	D_3	D_4	Supply
S_1	19**(5)**	30	50	10	2
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	0	8	7	14	

In 2nd column, The smallest transportation cost is 8 in cell S3D2

The allocation to this cell is min(18,8) = 8.

This satisfies the entire demand of D2 and leaves 18 - 8 = 10 units with S3
Table 2:

	D_1	D_2	D_3	D_4	Supply

S_1	19**(5)**	30	50	10	2
S_2	70	30	40	60	9
S_3	40	8**(8)**	70	20	10
Demand	0	0	7	14	

In 3rd column, The smallest transportation cost is 40 in cell S2D3

The allocation to this cell is min(9,7) = 7.

This satisfies the entire demand of D3 and leaves 9 - 7 = 2 units with S2

Table 3:

	D_1	D_2	D_3	D_4	Supply
S_1	19**(5)**	30	50	10	2
S_2	70	30	40**(7)**	60	2
S_3	40	8**(8)**	70	20	10
Demand	0	0	0	14	

In 4th column, The smallest transportation cost is 10 in cell S1D4

The allocation to this cell is min(2,14) = 2.

This exhausts the capacity of S1 and leaves 14 - 2 = 12 units with D4

Table 4:

	D_1	D_2	D_3	D_4	Supply
S_1	19**(5)**	30	50	10**(2)**	0
S_2	70	30	40**(7)**	60	2
S_3	40	8**(8)**	70	20	10
Demand	0	0	0	12	

in 4th column, The smallest transportation cost is 20 in cell S3D4

The allocation to this cell is min(10,12) = 10.

This exhausts the capacity of S3 and leaves 12 - 10 = 2 units with D4

Table 5:

	D_1	D_2	D_3	D_4	Supply
S_1	19(5)	30	50	10(2)	0
S_2	70	30	40(7)	60	2
S_3	40	8(8)	70	20(10)	0
Demand	0	0	0	2	

In 4th column, The smallest transportation cost is 60 in cell S2D4

The allocation to this cell is min(2,2) = 2.
Table 6:

	D_1	D_2	D_3	D_4	Supply
S_1	19(5)	30	50	10(2)	0
S_2	70	30	40(7)	60(2)	0
S_3	40	8(8)	70	20(10)	0
Demand	0	0	0	0	

Initial basic feasible solution

	D_1	D_2	D_3	D_4	Supply
S_1	19 (5)	30	50	10 (2)	7
S_2	70	30	40 (7)	60 (2)	9
S_3	40	8 (8)	70	20 (10)	18
Demand	5	8	7	14	

the minimum total transportation cost $=19\times5+10\times2+40\times7+60\times2+8\times8+20\times10=779$

Here, the number of allocated cells = 6 is equal to $m + n - 1 = 3 + 4 - 1 = 6$

∴ This solution is non-degenerate

Least Cost Method (LCM)

Step-1: Select the cell having minimum unit cost cij and allocate as much as possible, i.e. min(si,dj).
Step-2: a. Subtract this min value from supply si and demand dj.

b. If the supply si is 0, then cross (strike) that row and If the demand dj is 0 then cross (strike) that column.

c. If min unit cost cell is not unique, then select the cell where maximum allocation can be possible
Step-3: Repeact this steps for all uncrossed (unstriked) rows and columns until all supply and demand values are 0.

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	

Solution:

TOTAL number of supply constraints : 3

TOTAL number of demand constraints : 4
Problem Table is

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	

The smallest transportation cost is 8 in cell S3D2

The allocation to this cell is min(18,8) = 8.
This satisfies the entire demand of D2 and leaves 18 - 8 = 10 units with S3

Table 1:

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9

S_3	40	8**(8)**	70	20	10
Demand	5	0	7	14	

The smallest transportation cost is 10 in cell S1D4

The allocation to this cell is min(7,14) = 7.
This exhausts the capacity of S1 and leaves 14 - 7 = 7 units with D4
Table 2:

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10**(7)**	0
S_2	70	30	40	60	9
S_3	40	8**(8)**	70	20	10
Demand	5	0	7	7	

The smallest transportation cost is 20 in cell S3D4
The allocation to this cell is min(10,7) = 7.
This satisfies the entire demand of D4 and leaves 10 - 7 = 3 units with S3
Table 3:

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10**(7)**	0
S_2	70	30	40	60	9
S_3	40	8**(8)**	70	20**(7)**	3
Demand	5	0	7	0	

The smallest transportation cost is 40 in cell S2D3

The allocation to this cell is min(9,7) = 7.

This satisfies the entire demand of D3 and leaves 9 - 7 = 2 units with S2

Table 4:

	D_1	D_2	D_3	D_4	Supply

S_1	19	30	50	10(7)	0
S_2	70	30	40(7)	60	2
S_3	40	8(8)	70	20(7)	3
Demand	5	0	0	0	

The smallest transportation cost is 40 in cell S3D1
The allocation to this cell is min(3,5) = 3.
This exhausts the capacity of S3 and leaves 5 - 3 = 2 units with D1

Table 5:

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10(7)	0
S_2	70	30	40(7)	60	2
S_3	40(3)	8(8)	70	20(7)	0
Demand	2	0	0	0	

The smallest transportation cost is 70 in cell S2D1

The allocation to this cell is min(2,2) = 2.

Table 6:

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10(7)	0
S_2	70(2)	30	40(7)	60	0
S_3	40(3)	8(8)	70	20(7)	0
Demand	0	0	0	0	

Initial feasible solution is

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10 (7)	7

S_2	70 **(2)**	30	40 **(7)**	60	9
S_3	40 **(3)**	8 **(8)**	70	20 **(7)**	18
Demand	5	8	7	14	

The minimum total transportation cost $=10\times7+70\times2+40\times7+40\times3+8\times8+20\times7=814$

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6

∴ This solution is non-degenerate.

VOGELS APPROXIMATION METHOD

Procedure

Step-1: Find the cells having smallest and next to smallest cost in each row and write the difference (called penalty) along the side of the table in row penalty.

Step-2: Find the cells having smallest and next to smallest cost in each column and write the difference (called penalty) along the side of the table in each column penalty.

Step-3: Select the row or column with the maximum penalty and find cell that has least cost in selected row or column. Allocate as much as possible in this cell.

If there is a tie in the values of penalties then select the cell where maximum allocation can be possible

Step-4: Adjust the supply & demand and cross out (strike out) the satisfied row or column.

Step-5: Repeact this steps until all supply and demand values are 0.

Find Solution using Voggel's Approximation method

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	

Solution

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	

Table 1:

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19	30	50	10	7	9=19-10
S_2	70	30	40	60	9	10=40-30
S_3	40	8	70	20	18	12=20-8
Demand	5	8	7	14		
Column Penalty	21=40-19	22=30-8	10=50-40	10=20-10		

The maximum penalty, 22, occurs in column D2.
The minimum cij in this column is c32 = 8.
The maximum allocation in this cell is min(18,8) = 8.
It satisfy demand of D2 and adjust the supply of S3 from 18 to 10 (18 - 8 = 10).

Table 2:

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19	30	50	10	7	9=19-10
S_2	70	30	40	60	9	20=60-40
S_3	40	8**(8)**	70	20	10	20=40-20
Demand	5	0	7	14		
Column Penalty	21=40-19	--	10=50-40	10=20-10		

The maximum penalty, 21, occurs in column D1.
The minimum cij in this column is c11 = 19.
The maximum allocation in this cell is min(7,5) = 5.
It satisfy demand of D1 and adjust the supply of S1 from 7 to 2 (7 - 5 = 2).

Table 3:

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19**(5)**	30	50	10	2	40=50-10
S_2	70	30	40	60	9	20=60-40
S_3	40	8**(8)**	70	20	10	50=70-20
Demand	0	0	7	14		
Column Penalty	--	--	10=50-40	10=20-10		

The maximum penalty, 50, occurs in row S3.
The minimum cij in this row is c34 = 20.

The maximum allocation in this cell is min(10,14) = 10.
It satisfy supply of S3 and adjust the demand of D4 from 14 to 4 (14 - 10 = 4).

Table 4:

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19**(5)**	30	50	10	2	40=50-10
S_2	70	30	40	60	9	20=60-40
S_3	40	8**(8)**	70	20**(10)**	0	--
Demand	0	0	7	4		
Column Penalty	--	--	10=50-40	50=60-10		

The maximum penalty, 50, occurs in column D4.
The minimum cij in this column is c14 = 10.
The maximum allocation in this cell is min(2,4) = 2.
It satisfy supply of S1 and adjust the demand of D4 from 4 to 2 (4 - 2 = 2).

Table 5:

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19**(5)**	30	50	10**(2)**	0	--
S_2	70	30	40	60	9	20=60-40
S_3	40	8**(8)**	70	20**(10)**	0	--
Demand	0	0	7	2		
Column Penalty	--	--	40	60		

The maximum penalty, 60, occurs in column D4.
The minimum cij in this column is c24 = 60.
The maximum allocation in this cell is min(9,2) = 2.
It satisfy demand of D4 and adjust the supply of S2 from 9 to 7 (9 - 2 = 7).

Table 6:

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19**(5)**	30	50	10**(2)**	0	--
S_2	70	30	40	60**(2)**	7	40
S_3	40	8**(8)**	70	20**(10)**	0	--
Demand	0	0	7	0		
Column Penalty	--	--	40	--		

The maximum penalty, 40, occurs in row S2.

The minimum cij in this row is c23 = 40.

The maximum allocation in this cell is min(7,7) = 7.

It satisfy supply of S2 and demand of D3.

Initial basic feasible solution is

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19(5)	30	50	10(2)	7	9 | 9 | 40 | 40 | -- | -- |
S_2	70	30	40(7)	60(2)	9	10 | 20 | 20 | 20 | 20 | 40
S_3	40	8(8)	70	20(10)	18	12 | 20 | 50 | -- | -- | -- |
Demand	5	8	7	14		
Column Penalty	21 21 -- -- -- --	22 -- -- -- -- --	10 10 10 10 40 40	10 10 10 50 60 --		

The minimum total transportation cost $=19\times5+10\times2+40\times7+60\times2+8\times8+20\times10=779$
Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6
∴ This solution is non-degenerate

TEST OF OPTIMALITY

MODI METHOD

Procedure

Step-1: Find an initial basic feasible solution using any one of the three methods NWCM, LCM or VAM.

Step-2: Find ui and vj for rows and columns. To start

a. assign 0 to ui or vj where maximum number of allocation in a row or column respectively.

b. Calculate other ui's and vj's using cij=ui+vj, for all occupied cells.

Step-3: For all unoccupied cells, calculate dij=cij-(ui+vj), .

Step-4: Check the sign of dij

a. If dij>0, then current basic feasible solution is optimal and stop this procedure.

b. If dij=0 then alternative soluion exists, with different set allocation and same transportation cost. Now stop this procedure.

c. If dij<0, then the given solution is not an optimal solution and further improvement in the solution is possible.

Step-5: Select the unoccupied cell with the largest negative value of dij, and included in the next solution.

Step-6: Draw a closed path (or loop) from the unoccupied cell (selected in the previous step). The right angle turn in this path is allowed only at occupied cells and at the original unoccupied cell. Mark (+) and (-) sign alternatively at each corner, starting from the original unoccupied cell.

Step-7: 1. Select the minimum value from cells marked with (-) sign of the closed path.

2. Assign this value to selected unoccupied cell (So unoccupied cell becomes occupied cell).

3. Add this value to the other occupied cells marked with (+) sign.

4. Subtract this value to the other occupied cells marked with (-) sign.

Step-8: Repeat Step-2 to step-7 until optimal solution is obtained. This procedure stops when all dij≥0 for unoccupied cells.

Solve the following transportation problem.

	D1	D2	D3	D4	Supply
S1	11	13	17	14	250
S2	16	18	14	10	300
S3	21	24	13	10	400
Demand	200	225	275	250	

Initial basic feasible solution is obtained by using VAM method

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	11**(200)**	13**(50)**	17	14	250	2 | 1 | -- | -- | -- | -- |
S_2	16	18**(175)**	14	10**(125)**	300	4 | 4 | 4 | 4 | -- | -- |
S_3	21	24	13**(275)**	10**(125)**	400	3 | 3 | 3 | 3 | 3 | 10 |

Demand	200	225	275	250		
Column Penalty	5 -- -- -- -- --	5 5 6 -- -- --	1 1 1 1 13 --	0 0 0 0 10 10		

The minimum transportation cost=11×200+13×50+18×175+10×125+13×275+10×125=12075

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6

∴ This solution is non-degenerate

Optimality test using modi method...

Allocation Table is

	D_1	D_2	D_3	D_4	Supply
S_1	11 **(200)**	13 **(50)**	17	14	250
S_2	16	18 **(175)**	14	10 **(125)**	300
S_3	21	24	13 **(275)**	10 **(125)**	400
Demand	200	225	275	250	

Iteration-1 of optimality test

1. Find ui and vj for all occupied cells(i,j), where cij=ui+vj

1. Substituting, u1=0, we get

2. C11 =u1+v1⇒v1=c11-u1⇒v1=11-0⇒v1=11

3. C12 =u1+v2⇒v2=c12-u1⇒v2=13-0⇒v2=13

4. C22=u2+v2⇒u2=c22-v2⇒u2=18-13⇒u2=5

5. $C24=u2+v4 \Rightarrow v4=c24-u2 \Rightarrow v4=10-5 \Rightarrow v4=5$

6. $C34=u3+v4 \Rightarrow u3=c34-v4 \Rightarrow u3=10-5 \Rightarrow u3=5$

7. $C33=u3+v3 \Rightarrow v3=c33-u3 \Rightarrow v3=13-5 \Rightarrow v3=8$

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	11 **(200)**	13 **(50)**	17	14	250	$u_1=0$
S_2	16	18 **(175)**	14	10 **(125)**	300	$u_2=5$
S_3	21	24	13 **(275)**	10 **(125)**	400	$u_3=5$
Demand	200	225	275	250		
v_j	$v_1=11$	$v_2=13$	$v_3=8$	$v_4=5$		

Find dij for all unoccupied cells (i,j), where $dij=cij-(ui+vj)$

1.$d13=c13-(u1+v3)=17-(0+8)=9$

2.$d14=c14-(u1+v4)=14-(0+5)=9$

3.$d21=c21-(u2+v1)=16-(5+11)=0$

4.$d23=c23-(u2+v3)=14-(5+8)=1$

5.$d31=c31-(u3+v1)=21-(5+11)=5$

6.$d32=c32-(u3+v2)=24-(5+13)=6$

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	11 **(200)**	13 **(50)**	17 [9]	14 [9]	250	$u_1=0$
S_2	16 [0]	18 **(175)**	14 [1]	10 **(125)**	300	$u_2=5$
S_3	21 [5]	24 [6]	13 **(275)**	10 **(125)**	400	$u_3=5$
Demand	200	225	275	250		
v_j	$v_1=11$	$v_2=13$	$v_3=8$	$v_4=5$		

Since all dij≥0,final optimal solution is arrived.

	D_1	D_2	D_3	D_4	Supply
S_1	11 **(200)**	13 **(50)**	17	14	250
S_2	16	18 **(175)**	14	10 **(125)**	300
S_3	21	24	13 **(275)**	10 **(125)**	400
Demand	200	225	275	250	

The minimum total transportation cost $=11\times200+13\times50+18\times175+10\times125+13\times275+10\times125=12075$

UNBALANCED TRANSPORTATION PROBLEM

	D1	D2	D3	Supply
S1	**4**	**8**	**8**	**76**
S2	**16**	**24**	**16**	**82**
S3	**8**	**16**	**24**	**77**
Demand	**72**	**102**	**41**	

Here Total Demand = 215 is less than Total Supply = 235. So we add a dummy demand constraint with 0 unit cost and with allocation 20.

Now, The modified table is

	D1	D2	D3	D4(dummy)	Supply
S1	**4**	**8**	**8**	**0**	**76**
S2	**16**	**24**	**16**	**0**	**82**
S3	**8**	**16**	**24**	**0**	**77**
Demand	**72**	**102**	**41**	**20**	

Solution by NWCR method

The initial basic feasible solution is

	D_1	D_2	D_3	D_{4dummy}	Supply
S_1	4 **(72)**	8 **(4)**	8	0	76
S_2	16	24 **(82)**	16	0	82
S_3	8	16 **(16)**	24 **(41)**	0 **(20)**	77
Demand	72	102	41	20	

The total minimum transportation cost is=4×72+8×4+24×82+16×16+24×41+0×20=3528

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6

∴ This solution is non-degenerate

Solution by LCM method

The initial basic feasible solution is

	D_1	D_2	D_3	D_{4dummy}	Supply
S_1	4 **(56)**	8	8	0 **(20)**	76
S_2	16	24 **(41)**	16 **(41)**	0	82
S_3	8 **(16)**	16 **(61)**	24	0	77
Demand	72	102	41	20	

The minimum total transportation cost =4×56+0×20+24×41+16×41+8×16+16×61=2968

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6

∴ This solution is non-degenerate

Solution by VAM method

	D_1	D_2	D_3	D_{4dummy}	Supply	Row Penalty
S_1	4	8**(76)**	8	0	76	4 | 4 | -- | -- | -- | -- |
S_2	16	24**(21)**	16**(41)**	0**(20)**	82	16 | 0 | 0 | 8 | 24 | -- |
S_3	8**(72)**	16**(5)**	24	0	77	8 | 8 | 8 | 8 | 16 | 16 |
Demand	72	102	41	20		
Column Penalty	4 4 8 -- -- --	8 8 8 8 8 16	8 8 8 8 -- --	0 -- -- -- -- --		

The minimum total transportation cost =8×76+24×21+16×41+0×20+8×72+16×5=2424

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6

∴ This solution is non-degenerate

Assignment

An assignment problem is a particular case of transportation problem where the objective is to assign a number of resources to an equal number of activities so as to minimize total cost or maximize total profit of allocation. The problem of assignment arises because available resources such as men, machines etc. have varying degrees of efficiency for performing different activities, therefore, cost, profit or loss of performing the different activities is different.

Definition of Assignment Problem:

Suppose there are n jobs to be performed and n persons are available for doing these jobs. Assume that each person can do each job at a term, though with varying degree of efficiency, let c_{ij} be the cost if the i-th person is assigned to the j-th job. The problem is to find an assignment (which job should be assigned to which person one on-one basis) So that the total cost of performing all jobs is minimum, problem of this kind are known as assignment problem.

The following table represents the time taken by the n persons to perform the n jobs.

$$
\begin{array}{cc}
 & \textit{Jobs} \\
 & \begin{array}{ccccccc} \; & 1 & 2 & 3 & & j & n \end{array} \\
\text{Persons} & \left[\begin{array}{ccccccc}
1 & C_{11} & C_{12} & C_{13} & \cdots\cdots & C_{ij} \cdots & C_{1n} \\
2 & C_{21} & C_{22} & C_{23} & \cdots\cdots & C_{2j} \cdots & C_{2n} \\
3 & C_{31} & C_{32} & C_{33} & \cdots\cdots & C_{3j} \cdots & C_{3n} \\
\cdot & & & & & & \\
i & C_{11} & C_{12} \cdots & C_{13} & \cdots\cdots & C_{Cj} \cdots & C_{in} \\
n & C_{n1} & C_{n2} \cdots & C_{n3} & \cdots\cdots & C_{nj} \cdots & C_{nn}
\end{array}\right]
\end{array}
$$

Mathematically an assignment problem can be stated as follows

Minimise the total cost

$$Z = \sum_{i=i}^{n} \sum_{j=1}^{n} c_{ij} \cdot x_{ij}$$

where x_{ij} = 1, if i^{th} person is assgned to the j^{th} job

0, if i^{th} person is that assigned to the j^{th} job

subject to the constraints

(i) $\sum_{i=1}^{n} x_{ij} = 1, j = 1,2 \text{---} n$

which means that only one job is done by the i-th person, i =1,2 ---n

(ii) $\sum_{j=1}^{n} x_{ij} = 1, i = 1,2\, n$

which means that only one person should be assigned to the j^{th} job, j = 1,2 --- n

Procedure :

Step 1: Develop the Cost Table from the given Problem:

If the no of rows are not equal to the no of columns and vice versa, a dummy row or dummy column must be added. The assignment cost for dummy cells are always zero.

Step 2: Find the Opportunity Cost Table:

(a) Locate the smallest element in each row of the given cost table and then subtract that from each element of that row.

(b) In the reduced matrix obtained from 2 (a) locate the smallest element in each column and then subtract that from each element. Each row and column now have at least one zero value.

Step 3: Make Assignment in the Opportunity Cost Matrix:

The procedure of making assignment is as follows:

(a) Examine rows successively until a row with exactly one unmarked zero is obtained. Make an assignment single zero by making a square around it.

(b) For each zero value that becomes assigned, eliminate (Strike off) all other zeros in the same row and/ or column

(c) Repeat step 3 (a) and 3 (b) for each column also with exactly single zero value all that has not been assigned.

(d) If a row and/or column has two or more unmarked zeros and one cannot be chosen by inspection, then choose the assigned zero cell arbitrarily.

(e) Continue this process until all zeros in row column are either enclosed (Assigned) or struck off

Step 4: Optimality Criterion:

If the member of assigned cells is equal to the numbers of rows column then it is optimal solution. The total cost associated with this solution is obtained by adding original cost figures in the occupied cells.

If a zero cell was chosen arbitrarily in step (3), there exists an alternative optimal solution. But if no optimal solution is found, then go to step (5).

Step 5: Revise the Opportunity Cost Table:

Draw a set of horizontal and vertical lines to cover all the zeros in the revised cost table obtained from step (3), by using the following procedure:

(a) For each row in which no assignment was made, mark a tick (√)

(b) Examine the marked rows. If any zero occurs in those columns, tick the respective rows that contain those assigned zeros.

(c) Repeat this process until no more rows or columns can be marked.

(d) Draw a straight line through each marked column and each unmarked row.

If a no of lines drawn is equal to the no of (or columns) the current solution is the optimal solution, otherwise go to step 6.

Step 6: Develop the New Revised Opportunity Cost Table:

(a) From among the cells not covered by any line, choose the smallest element

(b) Subtract that element in the cell not covered by line.

(c) Add it to very element in the cell covered by the two lines, i.e., intersection of two lines.

(d) Elements in cells covered by one line remain unchanged.

Step 7: Repeat Step 3 to 6 Unlit an Optimal Solution is Obtained:

From the optimal solution for the assignment problem with the following cost matrix

Area

Salesman		W	X	Y	Z
	A	11	17	8	16
	B	9	7	12	6
	C	13	16	15	12
	D	14	10	12	11

SolutionStep1:

Choose the least element in each row and subtract it from all the elements of that row

	W	X	Y	Z
A	3	9	0	8
B	3	1	6	0
C	1	4	3	0
D	4	0	2	1

Step 2:

Choose the least element in each column and subtract it from all the elements of that column.

	W	X	Y	Z
A	2	9	[0]	8
B	2	1	6	[0]
C	[0]	4	3	0
D	3	[0]	2	1

The optimal assignment is given below

Salesman	Area	Cost
A	Y	8
B	Z	6
C	W	13
D	X	10
Total minimum cost		**37**

Solve the following assignment problem of minimizing total time for doing all the jobs(unbalanced assignment)

Job

Operator	1	2	3	4	5
1	6	2	5	2	6
2	2	5	8	7	7
3	7	8	6	9	8
4	6	2	3	4	5
5	9	3	8	9	7
6	4	7	4	6	8

Solution

This is an unbalanced assignment problem. Introduce a dummy job 6 with all cost zero.

	1	2	3	4	5	6
1	6	2	5	2	6	0
2	2	5	8	7	7	0
3	7	8	6	9	8	0
4	6	2	3	4	5	0
5	9	3	8	9	7	0
6	4	7	4	6	8	0

Step 1: The same table as above

Step 2:

4	0	2	0	1	0
0	3	5	5	2	0
5	6	3	7	3	0
4	0	0	2	0	0
7	1	5	7	2	0
2	5	1	4	3	0

No optimum solution is possible at this step.

Step 3:

5	0	2	0	1	0
0	2	4	4	1	0
5	5	2	6	2	0
5	0	0	2	0	1
7	0	4	6	1	0
2	4	0	3	2	0

Step4:

5	0	2	0	1	1
0	2	4	4	1	0
5	5	2	6	2	0
5	0	0	2	0	1
7	0	4	6	1	0
2	4	0	3	0	0

Step 5:

5	0	2	0	1	2
0	1	3	3	0	0
4	4	1	5	1	0
5	0	0	2	0	2
6	0	3	5	0	0
1	3	0	2	0	0

The optimal solution is given below

Operator	Job	time
1	4	2
2	1	2
3	6	0
4	5	5
5	2	3
6	3	4
Total minimum time required		16

SEQUENCING OF JOBS

Types of Sequencing Problems

There are various types of sequencing problems arise in real world. All sequencing problems cannot be solved. Though mathematicians and Operations Research scholars are working hard on the problem satisfactory method of solving problem is available for few cases only. The problems, which can be

solved, are:

(*a*) '*n*' jobs are to be processed on two machines say machine *A* and machine *B* in the order *AB*. This means that the job is to be processed first on machine *A* and then on machine *B*.

(*b*) '*n*' jobs are to be processed on three machines *A*,*B* and *C* in the order *ABC* *i.e.* first on machine *A*, second on machine B and third on machine *C*.

(*c*) '*n*' jobs are to be processed on '*m*' machines in the given order

(*d*) Two jobs are to be processed on '*m*' machines in the given order.

A machine operator has to perform two operations, turning and threading, on a number of different jobs. The time required to perform these operations in minutes for each job is given. Determine the order in which the jobs should be processed in order to minimize the total time required to turn out all the jobs.

Jobs	1	2	3	4	5	6	7
Time for turning(minutes)	8	10	10	6	12	1	3
Time for threading(minutes)	3	12	15	6	10	11	9

Solution

The smallest element is 1 in the given matrix and falls under first operation. Hence do the 6 th job first. Next smallest element is 3 for the job 1 and falls under second operation hence do the first job last. Like this go on proceed until all jobs are over. The optimal sequence is:

6	7	4	2	3	5	1

Job	Operation 1		Operation 2		Idle time	
	Time in	Time out	Time in	Time out	1	2
6	0	1	1	12	-	1
7	1	4	12	21	-	-
4	4	10	21	27	-	-
2	10	20	27	39	-	-
3	20	30	39	54	--	-
5	30	42	54	64	-	-
1	42	50	64	67	17	-

The minimum time required for all the jobs is 67 hours.

UNIT 3

INVENTORY MANAGEMENT

INTRODUCTION

Inventory management or Inventory Control is one of the techniques of Materials Management which helps the management to improve the productivity of capital by reducing the material costs, preventing the large amounts of capital being locked up for long periods, and improving the capital -turn over ratio. The techniques of inventory control were evolved and developed during and after the Second World War and have helped the more industrially developed countries to make spectacular progress in improving their productivity.

MEANING

The word *inventory* means a physical stock of material or goods or commodities or other economic resources that are stored or reserved or kept in stock or in hand for smooth and efficient running of future affairs of an organization at the minimum cost of funds or capital blocked form of materials or goods (Inventories).

TYPES:

1. Raw materials
2. Work-in-progress
3. Finished goods

Reasons for holding stock:

1. To ensure sufficient goods are available to meet anticipated demand.
2. To absorb variations in demand and production.
3. To take advantage of bulk productions.
4. To prove a buffer between production process
5. To meet possible shortages in the future.

COSTS ASSOCIATED WITH INVENTORY

While maintaining the inventories, we will come across certain costs associated with inventory, which are known as **economic parameters**.
(A) Inventory Carrying Charges, or Inventory Carrying Cost or Holding Cost or StorageCost ($C1$) or (i%):

The components of inventory carrying cost are:

(*i*) Rent for the building in which the stock is maintained if it is a rented building. In case it is own building, depreciation cost of the building is taken into consideration. Sometimes for own buildings, the nominal rent is calculated depending on the local rate of rent and is taken into consideration.

(*ii*) It includes the cost of equipment if any and cost of racks and any special facilities used in the stores.

(*iii*) Interest on the money locked in the form of inventory or on the money invested in purchasing the inventory.

(*iv*) The cost of stationery used for maintaining the inventory.

(*B*) Shortage cost or Stock - out - cost- (*C*2)

Sometimes it so happens that the material may not be available when needed or when the demand arises. In such cases the production has to be stopped until the procurement of the material, which may lead to miss the delivery dates or delayed production

***C*) Set up cost or Ordering cost or Replenishment Cost (*C*3)**

For purchase models, the cost is termed as ordering cost or procurement cost and for manufacturing cost it is termed as set up cost and is represented by *C*3.

(*i*) *Set up cost:* The term set up cost is used for production or manufacturing models. Whenever a job is to be produced, the machine is to set to produce the job. That is the tool is to be the material is to be fixed in the jobholder.

(*ii*) *Ordering Cost or Replenishment Cost: The* term Ordering cost or Replenishment cost is used in purchase models. Whenever any material is to be procured by an organization, it has to place an order with the supplier.

(*iii*) *Procurement Cost :* These costs are very much similar to the ordering cost / set up cost. This cost includes cost of inspection of materials, cost of returning the low quality materials, transportation cost from the source of material to the purchaser‘s site.

ABC Analysis of Inventory

This is sometimes known as Always Better Control. This system of control is also known as **System.** In *ABC* system of inventory control, the materials are classified depending on their turnover and **annual consumption cost.**

Q. No: 1

The annual demand for an item is 3200 units. The unit cost is Rs.6 and inventory carrying charges 25% per annum. If the cost of one procurement is Rs. 150, determine

i) Economic order quantity
ii) No. of orders per year
iii) Time between two consecutive orders
iv) The optimal cost

Solution

Annual demand (D) = 3200 units
Procurement cost (C_s) = Rs. 150/-
Inventory carrying cost (C_1) = Rs. $6 \times \frac{25}{100}$ = Rs. 1.50/-

i) $EOQ = Q^0 = \sqrt{\frac{2DC_s}{C_1}}$

$= \sqrt{\frac{2 \times 3200 \times 150}{1.50}} = 800$ units

ii) No. of orders per year (n) $= \frac{D}{Q^0} = \frac{3200}{800} = 4$

iii) Optimum time between two consecutive orders

$t_0 = \frac{Q^0}{D} = \frac{800}{3200} = \frac{1}{4}$ year or 3 months

iv) The optimal cost $\sqrt{2DC_1C_s}$

$= \sqrt{2 \times 3200 \times 1.50 \times 150}$

= Rs. 1200

Q. No: 2

An item is produced at the rate of 50 items per day. The demand occurs at the rate of 25 items per day. If the set up cost is Rs. 100 per set up and holiday cost is Rs. 0.01 per unit of item per day, find the economic lot size for one run assuming that the shortages are not allowed.

Solution:

$k = 50$ units per day

$r = 25$ units per day

$C_s = Rs.\ 100$

$C_1 = Rs.\ 0.01$

$$EOQ = Q^0 = \sqrt{\frac{2C_s}{C_1}\left(\frac{kr}{k-r}\right)}$$

$$= \sqrt{\frac{2\times 100}{0.01} \times \left(\frac{50\times 25}{50-25}\right)}$$

$$= \sqrt{\frac{2\times 100}{0.01}\left(\frac{50\times 25}{25}\right)}$$

$$= \sqrt{\frac{2\times 100}{0.01} \times 50}$$

$= 1000$ units

Q. NO: 3

Given the following data for an item of unit from demand, instaneous delivery time and back order facility.

Annual demand = 800 units
Cost of an item = Rs. 40
Ordering cost = Rs. 800
Inventory carrying cost = 40%
Back order cost = Rs. 10

Solution :

$D = 800$ units
$P = Rs. 40$
$C_s = Rs. 800$
$C_1 = Rs.\ 0.40 \times 40 = 16$
$C_2 = Rs. 10$

i) EOQ - $Q^0 = \sqrt{\frac{2DC_s}{C_1}\left(\frac{C_1+C_2}{C_2}\right)}$

$= \sqrt{\frac{2 \times 800 \times 800}{16}\left(\frac{16+10}{10}\right)} = 456$ units

ii) Maximum inventory level

$Q_1^0 = Q^0 \times \frac{C_2}{C_1+C_2}$

$= 456 \times \frac{10}{10+16}$

$= 175$

iii) Maximum number of back order quantity

$= Q^0 - Q_1^0$

$= 456 - 175$

$= 281$

iv) Time between orders $= t^0 = \frac{Q^0}{D}$

$= \frac{456}{800}$ year

$= 0.57$ year (or) 108 days

v) Total annual cost = Cost of the material + Variable inventory cost

$= 800 + 40 \times \sqrt{2DC_1C_s\left(\frac{C_2}{C_1+C_2}\right)}$

$= 800 + 40 \times \sqrt{2 \times 800 \times 16 \times 800\left(\frac{10}{10+10}\right)}$

$= 32000 + 2807$

$=$ Rs. 34,807/-

Q. No: 4

Find the optimal order quantity for a product for which the price breaks are as follows:

Quantity	Purchasing Cost
$0 \le Q_1 < 100$	Rs. 20 per unit
$100 \le Q_2 < 200$	Rs. 18 per unit
$200 \le Q_3$	Rs. 16 per unit

The monthly demand for the product is 400 units. The storage cost is 20% of the unit cost of the product per month and the cost of ordering is Rs. 25.00

Solution

$D = 400$ units per month

C_s = Rs. 25

C_1 = 20% of unit price per month

$$Q_3^0 = \sqrt{\frac{2DC_s}{C_1}} = \sqrt{\frac{2 \times 400 \times 25}{0.20 \times 16}} = 79 \text{ units}$$

Q_3^0 does not lie in the last range

Q_3^0 cannot be the EOQ

$$Q_2^0 = \sqrt{\frac{2 \times 400 \times 25}{0.20 \times 20}} = 70 \text{ units}$$

Now we have to compare $C_A(Q_1^0)$, $C_A(100)$ and $C_A(200)$ to find the real EOQ

$C_A(Q_1^0)$ = Cost of the material + Total inventory cost

$= 400 \times 20 + \sqrt{2DC_1C_s}$

$= 8000 + \sqrt{2 \times 400 \times 4 \times 25}$

$= 8000 + 282.84$

= Rs. 8283 nearly

$$C_A(100) = 400 \times 18 + \frac{400}{100} \times 25 + \frac{1}{2} \times 100 \times 3.6$$

$$= 7200 + 100 + 180$$

$$= \text{Rs. } 7480$$

$$C_A(200) = 400 \times 16 + \frac{400}{100} \times 25 + \frac{1}{2} \times 200 \times 3.2$$

$$= 6400 + 50 + 320$$

$$= \text{Rs. } 6770 /-$$

Since the total cost is minimum for 200 units.

Scanned with CamScanner

UNIT 4

SIMULATION

INTRODUCTION

Simulation is the most important technique used in analyzing a number of complex systems. There are many real world problems which cannot be represented by a mathematical model due to stochastic nature of the problem, the complexity in problem formulation and many values of the variables are not known in advance and there is no easy way to find these values. Simulation has become an important tool for tackling the complicated problem of managerial decision-making. Simulation determines the effect of a number of alternate policies without disturbing the real system.

Definition

According to Donald G. Malcolm, simulation model may be defined as one which depicts the working of a large scale system of men, machines, materials and information a period of time in a simulated environment of the actual real world conditions.

CLASSIFICATION OF SIMULATION MODELS

- Simulation of Deterministic models:

In the case of these models, the input and output variables are not permitted to be randomvariables and models are described by exact functional relationship.

- Simulation of Probabilistic models:

In such cases method of random sampling is used. The techniques used for solving thesemodels are termed as Monte-Carlo technique.

- Simulation of Static Models:

These models do not take variable time into consideration.

- Simulation of Dynamic Models:

These models deal with time varying interaction.

ADVANTAGES OF SIMULATION

- It is straightforward and flexible.
- It can be used to analyze large and complex real world situations that cannot be solved by conventional quantitative analysis models.
- It is the only method sometimes available.

- It studies the interactive effect of individual components or variables in order to determine which ones are important.
- Simulation model, once constructed, may be used over and again to analyze all kinds of different situations.

LIMITATIONS OF SIMULATION TECHNIQUE

- Since simulation model mostly deals with uncertainties, the results of simulation are only
- In many situations, it is not possible to identify all the variables, which affect the behaviour of the system.
- In very large and complex problems, it is very difficult to make the computer program in
- view of the large number of variables and the involved inter–relationship among them.

MONTE-CARLO SIMULATION

The Monte-Carlo method is a simulation technique in which statistical distribution functions are created using a series of random numbers. Working on the digital computer for a few minutes we can create data for months or years. The method is generally used to solve problems which cannot be adequately represented by mathematical models or where solution of the model is not possible by analytical method.

Monte-Carlo simulation yields a solution, which should be very close to the optimal, but not necessarily the exact solution. But this technique yields a solution, which converges to the optimal solution as the number of simulated trials tends to infinity. The Monte-Carlo simulation procedure can be summarized in the following steps:

Step 1: Clearly define the problem:

- Identify the objectives of the problem.
- Identify the main factors, which have the greatest effect on the objective of the problem.
- Step 2: Construct an approximate model:
- Specify the variables and parameters of the mode.
- Formulate the appropriate decision rules, i.e. state the conditions under which the experiment is to be performed.
- Identify the type of distribution that will be used. Models use either theoretical Distributions or empirical distributions to state the patterns of the occurrence associated with the variables

- Specify the manner in which time will change.

Q.No: 1

A Bakery store keeps stock of a popular brand of cake. The daily demand based on the past experience is given below:

Daily Demand	0	15	25	35	45	55
Probability	0.01	0.15	0.20	0.50	0.12	0.02

Using the following random numbers 48, 78, 09, 51, 56, 77, 15, 14, 68, 09. Simulate the demand for the next days. Find out the stock situation if the owner of the bakery decides to make 35 cakes every day. Also estimate the daily average demand for the cakes on the basis of simulated data.

Solution:

The following table shows the cumulative probabilities and the assigned random numbers.

Daily demand	Probability	Cum. Probability	Random No. assigned
0	0.01	0.01	00-00
15	0.15	0.16	01-15
25	0.20	0.36	16-35
35	0.50	0.86	36-85
45	0.12	0.98	86-97
55	0.02	1.00	98-99

The simulated demand is shown.

Day	Random No.	Simulated Demand	Stock situation
1	48	35	NIL
2	78	35	NIL
3	09	15	20
4	51	35	20
5	56	35	20
6	77	35	20
7	15	15	40
8	14	15	60
9	68	35	60
10	09	15	80

Expected demand = 270/10 = 27 cakes per day.

QUEUEING

INTRODUCTION

Before going to ***waiting line theory or queuing theory***, one has to understand two things in clear. They are ***service and customer or element.*** Here customer or element represents a person or machine or any other thing, which is in need of some service from servicing point. Service represents any type of attention to the customer to satisfy his need. For example,

1. Person going to hospital to get medical advice from the doctor is an element or a customer,
2. A person going to railway station or a bus station to purchase a ticket for the journey is a customer or an element,
3. A person at ticket counter of a cinema hall is an element or a customer,
4. A person at a grocery shop to purchase consumables is an element or a customer,
5. A bank pass book tendered to a bank clerk for withdrawal of money is an element or a customer,

STEADY, TRANSIENT AND EXPLOSIVE STATES IN A QUEUE SYSTEM

The distribution of customers arrival time and service time are the two constituents, which constitutes of study of waiting line. Under a fixed condition of customer arrivals and service facility a queue length is a function of time. As such a queue system can be considered as some sort of random experiment and the various events of the experiment can be taken to be various changes occurring in the system at any time. We can identify three states of nature in case of arrivals in a queue system. They are named as ***steady state, transient state, and the explosive state.***

(a) *Steady State:* The system will settle down as steady state when the rate of arrivals of customers is less than the rate of service and both are constant. The system not only becomes steady state but also becomes independent of the initial state of the queue.

(b) *Transient State:* Queuing theory analysis involves the study of system behaviour over time. A system is said to be in transient state when its operating characteristics or behaviour are dependent on time.

b) Explosive State: In a situation, where arrival rate of the system is larger than its service rate, a steady state cannot be reached regardless of the length of the elapsed time.

The following symbols are used in this model

m = number of customers in the waiting line

n = number of customers in the system.

w = waiting time of a customer in the queue

v = waiting time of a customer in the system.

λ = mean arrival rate

μ = mean service rate

ρ = capacity utilisation

1) $E(m) = \frac{\lambda^2}{\mu(\mu-\lambda)}$

2) $E(n) = \frac{\lambda}{\mu-\lambda}$

3) $E(w) = \frac{\lambda}{\mu(\mu-\lambda)}$

4) $E(v) = \frac{1}{\mu-\lambda}$

5) $\rho = \frac{\lambda}{\mu}$

6) $P(n>k)$ = The probability that there are more than k customers in the system

$$= \left(\frac{\lambda}{\mu}\right)^{k+1}$$

In particular the probability that there is atleast one customer in the system.

$$= P(n>0) = \frac{\lambda}{\mu}$$

Also the probability that there is no customer in the system.

$$= 1 - P(n>0)$$

$$= 1 - \frac{\lambda}{\mu}$$

Q.No:1

A TV repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repair sets in the order in which they come in and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hour day.

i) how many jobs are ahead of the set just brought in?

ii) What is the repairman's expected idle time each day?

Solution:

λ = mean arrival rate = $10/8$ per hour

μ = mean service rate = $60/30 = 2$ per hour

i) Average number of sets ahead of the one just brought in

= Average number of sets in the system

$$E(n) = \frac{\lambda}{\mu-\lambda}$$

$$= \frac{10/8}{2-10/8} = 5/3$$

ii) Time taken to repair one set = $\frac{1}{2}$ hour

No. of sets arriving per day = 10

Time taken to repair 10 sets = $10 \times \frac{1}{2} = 5$ hrs

No. of working hours per day = 8

$\therefore$ Idle time per day = $8 - 5$

= 3 hours.

Q. No: 2

Westem National Bank is considering opening a drive-in-window for customer service. Management estimates that the customers will arrive for service at the rate of 15 per hour. The teller whom it is considering to staff the window can service customer at the rate of one every three minutes. Assuming Poisson arrivals and exponential service time find i) Utilisation of the teller ii) average no. of customer in the waiting line iii) average number in the system iv) average waiting time in the line v) average waiting time in the system.

Solution

$$\lambda = 15 \text{ per hour}$$

$$\mu = 60/3 = 20 \text{ per hour}$$

i) Utilisation of the teller

$$\rho = \frac{\lambda}{\mu} = \frac{15}{3} = \frac{3}{4}$$

ii) Average number in the waiting line

$$E(m) = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{15^2}{20(20-15)} = 2.25 \text{ customer}$$

iii) Average number in the system

$$E(n) = \frac{\lambda}{\mu-\lambda} = \frac{15}{20-15} = 3 \text{ customers}$$

iv) Average waiting time in the line

$$E(w) = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{15}{20(20-15)} = \frac{3}{20} \text{ hr} = 9 \text{ minutes}$$

v) Average waiting time in the system

$$E(v) = \frac{\lambda}{\mu-\lambda} = \frac{1}{20-15} = \frac{1}{5} \text{ hour or 12 minutes.}$$

UNIT 5

NETWORK ANALYSIS

Network analysis is a technique which determines the various sequences of jobs concerning a project and the project completion time. Network analysis has been successfully used to a wide range of significant management problems. Programme Evaluation and Review Technique (PERT) and Critical Path Method (CPM) are two techniques that are widely used in planning and scheduling the large projects. A project is a combination of various activities.

The critical path method (CPM) was developed in 1957 by J.E.Kelly of Remington and Rand and M.R.Walker of Du pont to help schedule maintenance of chemical plants. This method differentiates between planning and scheduling. Planning refers to the determination of activities and the order in which such activities to be performed to achieve the objective of the project.

PERT Technique was developed in 1950s by the naval special projects office in co-operation with Booz, Allen and Hamilton, a management consulting firm. Using this technique the management was able to obtain the expected project completion time, and the bottleneck activities in a project. Generally we assume that the time to perform each activity is uncertain and that three types of time estimates are calculated.

(*a*) OPTIMISTIC TIME: Optimistic time is represented by **To**. Here the estimator thinks that everything goes on well and he will not come across any sort of uncertainties and estimates lowest time as far as possible. He is optimistic in his thinking.

(*b*) PESSIMISTIC TIME: This is represented by **Tp**. Here estimator thinks that everything goes wrong and expects all sorts of uncertainties and estimates highest possible time.

(*c*) LIKELY TIME: This is represented by ***Tl***. This time is in between optimistic and pessimistic times. Here the estimator expects he may come across some sort of uncertainties and many a time the things will go right.

Using these time estimates another time estimate called expected time estimate(Te) is obtained.

Te = (To+Tp+4Tm)/6

Standard deviation = Tp-To/6

Definitions

The earliest start time (EST): It is the earliest possible time at which an activity can be started assuming that the preceding activities can be s latest possible time started at the earliest start times.

Earliest finish time (EFT): The earliest finish time of an activity is the earliest start time plus the duration of the activity.

Latest finish time (LFT): The latest finish time of an activity is the latest possible time at which the activity can be finished assuming that all the subsequent activities can also be finished at the latest possible times.

The latest start time (LST): The latest start time of an activity is equal to the latest finish time minus the duration of the activity

Total float (TF): This is the amount of time a path of activities could be delayed without affecting the overall project duration.

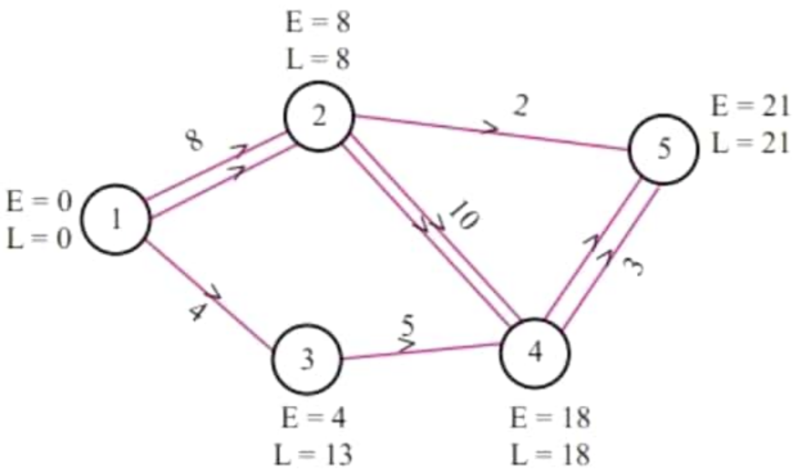
TF = LFT - EFT = LST – EST

Free float (FF): This is the amount of activity can be delayed without affecting the commencement of a subsequent activity at its earliest start time, but may affect the float of a previous activity.

Q.No:1

Compute the earliest start time, earliest finish time ,latest start time and latest finish time of each activity of the project given below:

Activity	1-2	1-3	2-4	2-5	3-4	4-5
Duration(in days)	8	4	10	2	5	3



Calculations

$E_1 = 0$

$E_2 = E_1 + t_{12} = 0 + 8 = 8$

$E_3 = E_1 + t_{13} = 0 + 4 = 4$

$E_4 = E_2 + t_{24}$ or $E_3 + t_{34} = 8 + 10 = 18$

(take $E_2 + t_{24}$ or $E_3 + t_{34}$ whichever is maximum)

$E_5 = (E_2 + t_{25}$ or $E_4 + t_{45}) = 18 + 3 = 21$

(take $E_2 + t_{25}$ or $E_4 + t_{45}$ whichever is maximum)

$L_5 = 21$

$L_4 = L_5 - t_{45} = 21 - 3 = 18$

$L_3 = L_4 - t_{34} = 18 - 5 = 13$

$L_2 = L_5 - t_{25}$ or $L_4 - t_{24} = 18 - 10 = 8$

(take $L_5 - t_{25}$ or $L_4 - t_{24}$ whichever is minimum)

$L_1 = L_2 - t_{12}$ or $L_3 - t_{13} = 8 - 8 = 0$

(take $L_2 - t_{12}$ or $L_3 - t_{13}$ whichever is minimum)

Here the critical path is 1-2-4-5, which is denoted by double lines.

Activity	Duration (t_{ij})	EST	EFT=EST+t_{ij}	LST=LFT−t_{ij}	LFT
1–2	8	0	8	0	8
1–3	4	0	4	9	13
2–4	10	8	18	8	18
2–5	2	8	10	19	21
3–4	5	4	9	13	18
4–5	3	18	21	18	21

The longest duration to complete this project is 21 days.

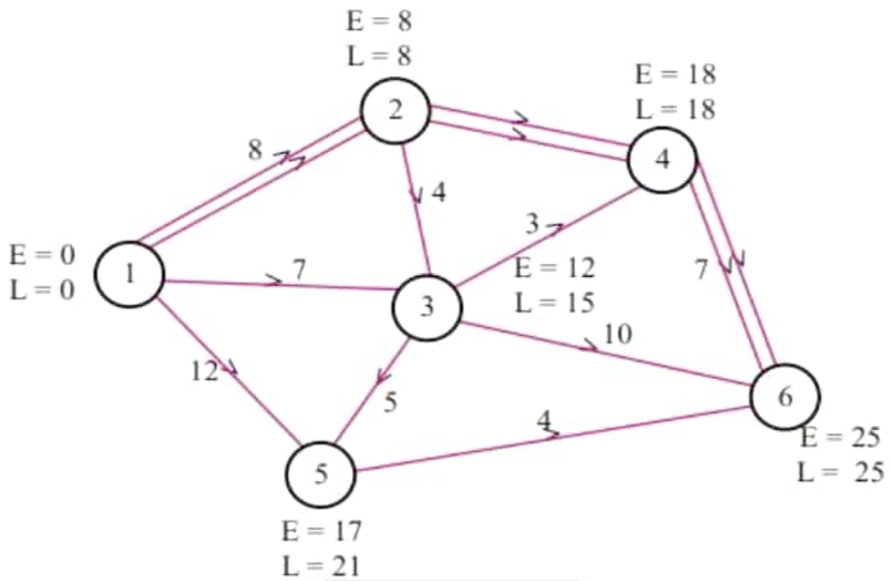
The path connected by the critical activities is the critical path (the longest path).

Critical path is 1-2-4-5 and project completion time is 21 days.

Q.No:2

Calculate the earliest start time, earliest finish time, latest start time and latest finish time of each activity of the project given below and determine the Critical path of the project and duration to complete the project.

Activity	1-2	1-3	1-5	2-3	2-4	3-4	3-5	3-6	4-6	5-6
Duration (in week)	8	7	12	4	10	3	5	10	7	4



Table

Activity	Duration (in week)	EST	EFT	LST	LFT
1-2	8	0	8	0	8
1-3	7	0	7	8	15
1-5	12	0	12	9	21
2-3	4	8	12	11	15
2-4	10	8	18	8	18
3-4	3	12	15	15	18
3-5	5	12	17	16	21
3-6	10	12	22	15	25
4-6	7	18	25	18	25
5-6	4	17	21	21	25

Here the critical path is 1–2–4–6

The project completion time is 25 weeks

Q. No: 3

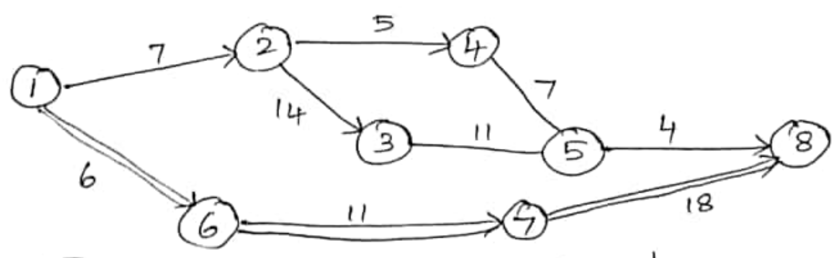
The following table lists the jobs of a network with their time estimates.

Job	Optimistic	Duration days Most likely	Pessimistic
1-2	3	6	15
1-6	2	5	14
2-3	6	12	30
2-4	2	5	8
3-5	5	11	17
4-5	3	6	15
6-7	3	9	27
5-8	1	4	7
7-8	4	19	8

Draw the project network and calculate the length and Variance of the critical path.

Solution

Job	$T_E = \frac{T_o + T_p + 4T_m}{6}$	$SD\left(\frac{T_p - T_o}{6}\right)$	Variance
1-2	7	2	4
1-6	6	2	4
2-3	12	4	16
2-4	5	1	1
3-5	11	2	4
4-5	7	2	4
6-7	11	4	16
5-8	4	1	1
7-8	18	4	16



The project duration is 36 days

The Variance of the critical path = 4+16+16 = 36

REPLACEMENT MODEL

The problem of replacement arises when any one of the components of productive resources, such as machinery, building and men deteriorates due to time or usage. The examples are:

(a) A machine, which is purchased and installed in a production system, due to usage some of its components wear out and its efficiency is reduced.

(b) A building in which production activities are carried out, may leave cracks in walls, roof etc, and needs repair.

(c) A worker, when he is young, will work efficiently, as the time passes becomes old and his work efficiency falls down and after some time he will become unable to work.

TYPES OF FAILURE

(i) Gradual failure

(ii) Sudden failure.

(a) Progressive failure, (b) Retrogressive failure and (c) Random failure.

Following table gives the running costs per year and resale price of certain equipment whose purchase price is Rs.5000

Year	1	2	3	4	5	6	7	8
Running cost(Rs.)	1500	1600	1800	2100	2500	2900	3400	4000
Resale value (Rs.)	3500	2500	1700	1200	800	500	500	500

At what year is the replacement due?

Solution

Year	Running cost(Rs.)	cumulative Running cost(Rs.)	Resale value (Rs.)	Cost-Resale value (Rs.)	T	T_a=T/Year
1	1500	1500	3500	1500	3000	3000
2	1600	3100	2500	2500	5600	2800
3	1800	4900	1700	3300	8200	2733
4	2100	7000	1200	3800	10800	2700
5	2500	9500	800	4200	13700	2740
6	2900	12400	500	4500	16900	2817
7	3400	15800	500	4500	20300	2900
8	4000	19800	500	4500	24300	3038

The average cost reaches its minimum value at the end of 4^{th} year. Hence the equipment should be replaced at the end of every 4^{th} year.

****